

EFFICIENT MOVEMENT OF FRAGMENT STAMP

RELATED APPLICATION

5 The present application is related to co-pending United States patent application bearing attorney docket number 9772-0019-999, entitled "METHOD AND APPARATUS FOR TILED POLYGON TRAVERSAL," filed August 20, 2001, which is hereby incorporated by reference in its entirety.

FIELD OF THE INVENTION

10 This invention relates generally to graphics accelerators, and more particularly to graphics accelerators that use half-plane edge functions to determine whether a given (x, y) position of a pixel is within a graphic object such as a line or triangle while rendering the object.

BACKGROUND OF THE INVENTION

Fragments

15 A three-dimensional (3D) graphic processing device uses a description of an object such as a polygon, line, or triangle to generate the object's constituent fragments. A fragment is defined as all information required to render a single pixel that is within the boundaries of the object, for example, the x and y coordinates of the pixel, the red, green and blue color values used to modify the pixel, alpha transparency and Z depth values, texture coordinates, and the like. The graphics device must determine which fragments are contained within the
20
25 object.

Half-Plane Edge Functions

30 A half-plane edge function fragment generator uses planar (affine) edge functions of the x and y screen coordinates. The values of these edge functions at a given pixel determine directly if the pixel is inside or outside an object, and so if a fragment should be generated for the pixel. An antialiased graphics device may evaluate the edge functions at several positions within a pixel to determine with greater precision which portions of a pixel are inside an object. Given the value of the edge functions at various points surrounding the
35 current position, the fragment generator decides where to go next.

An introduction to half-plane edge functions is given by J. Pineda in “A Parallel Algorithm for Polygon Rasterization”, *ACM Computer Graphics*, Volume 22, Number 4, August 1988 (SIGGRAPH 1998 issue), incorporated by reference herein, though the basic traversals methods described by Pineda are less than optimal.

5

As a very brief summary, each directed edge of an object, such as a triangle with three edges or a line with four edges, is represented as a function that partitions the 2D (x, y) rendering plane into two portions: at points to the left of the parting edge with respect to its direction, the function is negative, and at points on the parting edge or to the right of the parting edge the function is nonnegative, that is, zero, or positive.

By combining information from all edge functions at a given point, it can be determined whether the point is inside or outside the object. For example, if the three directed edges of a triangle connect in a clockwise fashion, then a point is inside the triangle if all three edge functions are nonnegative. If the three edges connect in a counterclockwise fashion, then a point is inside the triangle if all three edge functions are negative. Note that points along an edge or vertex that is shared between two or more objects should be assigned to exactly one object. The edge equations can be adjusted during setup to accomplish this.

Fig. 2 shows a triangle 200 that can be described by three clockwise directed edges 201-203, which are shown as bold arrows. The half-plane where each corresponding edge function is nonnegative is shown by the several thin “shadow” lines 210. Each shadow line 210 has the same slope as the corresponding edge. The shaded portion of Fig. 2 shows the area where all edge functions are nonnegative, i.e., points within the triangle object 200.

25

Fragment Stamp

One advantage of using half-plane equations is that parallel fragment generation is possible. For example, one can define a “fragment stamp” as a rectangle of *stampWidth* pixels by *stampHeight* pixels, and simultaneously compute all fragments that are within both the stamp and the object. Although fragment stamps of arbitrary size can be implemented, many computations involving the stamp can be performed with substantially less hardware if the stamp rectangle is 2^m pixels wide by 2^n pixels high. In prior art implementations, the stamp rectangle and probe points (discussed in more detail below) are essentially synonymous—the probes are located at the vertices of the stamp rectangle. More generally, though, a stamp includes three components: a rectangle, a set of sample points at which the

35

edge functions are evaluated for generating fragment information, and a set of probe points at which the edge functions are evaluated for moving the stamp to a new location. The set of sample points and probes may overlap, that is, at some points the edge functions are evaluated both for generating fragment information and for moving the stamp.

5

Prior Art Traversal Algorithms

The prior art stamp movement algorithms sketched by Pineda sweep the stamp horizontally left, then right across a row “stampline,” then step up or down somewhere into the next stampline. A row stampline is similar to a scanline (a row of pixels), except that a row stampline has a height equal to the height of the fragment stamp. Alternatively, the stamp can be moved vertically up and down in a column stampline, followed by stepping horizontally into the next column stampline. In this alternative, the column stampline has a width equal to the width of the fragment stamp.

10

The algorithms sketched by Pineda frequently allow the stamp to move outside of the object. This means that the stamp visits *unproductive* positions, for which it generates no fragments, then has to find its way back into the object. Thus, the Pineda algorithms frequently require many more cycles to traverse the object than an algorithm that avoids such unproductive positions.

15

20

Stamp Contexts

All traversal methods using half-plane functions require maintaining stamp *contexts*. A stamp context is all the information needed to place the stamp at a given position within the object. The context information includes the *x* and *y* position of the stamp, the value of all four half-plane edge evaluators, as well as the value of all *channel* data being interpolated from values provided at the object’s vertices. The channel data includes, for example, color, transparency, *Z* depth, and texture coordinates.

25

30

Determining Valid Nearby Stamp Positions

Pineda does not describe stamp movement algorithms in sufficient detail to determine what data are used to move the stamp. In particular, Pineda does not describe how to determine positions near the current stamp position to which the stamp might be moved. The stamp might move to such positions either immediately for evaluation during the next cycle, or farther in the future by saving the position in a stamp context, and eventually restoring the

35

saved context into the current stamp position. B. Kelleher provides greater detail about determining such positions for traversal algorithms in *PixelVision Architecture*, Technical Note 1998-013, System Research Center, Compaq Computer Corporation, October 1998, available at

5 <http://gatekeeper.dec.com/pub/DEC/SRC/technical-notes/abstracts/src-tn-1998-013.html>.

To summarize the PixelVision hardware, one corner of the fragment stamp is designated the *origin*, at the stamp-relative location (0, 0). The inventors therein arbitrarily make the upper left corner the origin, with increasing x values farther to the right, and increasing y values
10 farther down. The stamp is also augmented with three *probes*. The term *probe* is not used in the PixelVision document. The term “probe” herein refers to an (x, y) location at which the half-plane equations are evaluated to assist stamp movement, rather than to determine whether a pixel (or portion of a pixel) is within the object. The probes are located at stamp-
relative positions (*stampWidth*, 0), (0, *stampHeight*), and (*stampWidth*, *stampHeight*).
15 These will be referred to as *RIGHT_TOP* (RT), *LOWER_LEFT* (LL), and *RIGHT_BOTTOM* (RB).

Fig. 3 shows a stamp 300 that has a rectangle 4 pixels wide by 2 pixels high. The thin lines
310 are a grid of pixels. The stamp rectangle is shown with thick solid lines. The stamp
origin 320, used both for determining if the upper left pixel is in the object, and for assisting
stamp movement, is circled. The other sample points 330 are used strictly for determining
if the associated pixels are in the object, are shown with an X. The probes, used strictly to
assist stamp movement, are enclosed in diamonds, and are labeled *RT*, *LT*, and *RB*. Note
that the stamp rectangle’s edge segments are defined as (*ORIGIN*, *RT*), (*RT*, *RB*), (*RB*, *LL*),
20 and (*LL*, *ORIGIN*).
25

The movement algorithm tests each rectangle edge segment of the stamp to see if the edge
segment *intersects* the object. A stamp rectangle edge segment intersects the object if any
point along the segment is inside the object.

30 Testing for strict mathematical intersection of an edge segment with the object is computationally difficult. Instead, PixelVision probabilistically computes intersection semantics using two tests, which may yield a *false positive*. That is, the tests might
sometimes indicate an intersection when an edge segment does not truly intersect the object.
35

The first test computes if, for each of the three or four half-plane equations used to surround a triangle or quadrilateral, at least one of the two probes at the ends of the stamp edge segment is on the inside edge of the half-plane equation. Note that this does not require that the *same* probe be inside each of the three or four half-plane equations, only that *either* or *both* probes be inside each half-plane equation.

Fig. 4 shows a stamp 410 and a triangle 420. The first test indicates that the object intersects the top stamp rectangle edge segment (*ORIGIN*, *RT*) because *ORIGIN* and *RT* are inside *E0* and *E1*, and *ORIGIN* is inside *E2*. The first test indicates that the object intersects the right stamp rectangle edge segment (*RT*, *RB*) because *RT* is inside *E0*, both *RT* and *RB* are inside *E1*, and *RB* is inside *E2*. Likewise, the first test indicates that the object intersects the bottom and left stamp rectangle edge segments. It is easy to see that this first test will be true if a stamp rectangle edge segment intersects the object.

However this first test is also true if a stamp rectangle edge segment *spans* a portion of the object's *shadow*. The object's shadow is the set of points that are outside two edges, but inside the remaining edge(s). A stamp rectangle edge segment spans the shadow if the segment's endpoints are not in the shadow, but some point between the endpoints is in the shadow. The shaded portion of Fig. 5 shows a triangle object 500 and its shadow 510. Fig. 6 shows a triangle 610 for which the right edge segment (*RT*, *RB*) of stamp 600 satisfies the first intersection test: *RT* is inside *E0*, both *RT* and *RB* are inside *E1*, and *RB* is inside *E2*. However, the right edge segment of stamp 600 does not truly intersect triangle 610.

Thus, the first test for intersection with a stamp rectangle edge segment is augmented with a second test that ensures that the stamp rectangle edge segment is inside the minimal rectangular bounding box of the object, where the bounding box's edges are horizontal and vertical. If both these tests are true, then the stamp rectangle edge segment *probably* intersects the object. This second test eliminates the problem of indicating a false intersection for shadows associated with an object vertex that lies on one side of the minimal bounding box.

Note that object vertices that do not lie on an edge of the bounding box can still cast a false positive shadow. This does not cause any correctness problems—all productive stamp positions that intersect the object will still be visited. It does cause efficiency problems, as the false positive shadow intersection may cause the stamp to move outside the object and

thus waste a cycle generating no fragments. Fortunately, efficiency suffers insignificantly. For most objects, such deceitful vertices can only occur when two edges join at an obtuse (greater than 90°) angle at some point interior to the minimal bounding box, and in such a way that the shadow cannot be spanned by horizontal and vertical stamp edge segments.

5 For a few objects, such as X11 wide lines and OpenGL antialiased lines, computing an exact bounding box is difficult, and a graphics accelerator may instead compute a slightly larger bounding box. In these cases all four vertices of the line are inside the bounding box and thus cast a deceitful shadow. But even for these objects, the line edges meet at a 90° angle, and the shadow quickly grows beyond the size of a stamp edge segment, which
10 can then no longer span the shadow. In nearly all cases, the stamp will move at most one position outside the object.

Moving the stamp using information only from its rectangle's four edge segments results in moving to many more positions than necessary. Often, a stamp rectangle edge segment may
15 intersect the object, and yet moving the stamp in that direction is futile, as none of the sample points at that adjacent stamp position could possibly be within the object. For example, the triangle 420 in Fig. 4 intersects all four edges of the stamp rectangle 410, yet *none* of the four stamp positions above, below, left, or right of the current stamp position contains any sample points that are within the object. Such positions, for which the stamp's *stampWidth* by *stampHeight* pixel rectangle intersects the object, but none of the stamp's
20 sample points are within the object, are called *unproductive*.

It would be desirable to avoid visiting as many such unproductive stamp positions as possible with a reasonable implementation cost.

SUMMARY OF THE INVENTION

The present invention relates to a method and a computer system for visiting all productive stamp locations for a two-dimensional convex polygonal object, such as might be
30 encountered when rendering an object on a display device. The object is visited with a rectangular stamp, which contains one or more discrete sample points. A productive location is one in which the object contains at least one of the stamp's sample points when the stamp is placed at that location. An unproductive position is one in which the object contains none of the stamp's sample points. Stamp locations are discrete points that are
35 separated vertically by the stamp's height, and horizontally by the stamp's width. The stamp may move to a nearby position, or to a previously saved position, as it traverses the

object. Embodiments of the invention often avoid moving the stamp to unproductive positions.

5 In terms of the method, the invention uses each pair of vertices, in the order presented, to construct a directed edge between the vertices. Each directed edge is represented by an affine function of the form $E(x, y) = Ax + By + C$, in which all points to the left of the edge have a negative value, all points on the edge have a zero value, and all points to the right of the edge have a positive value. Points are considered within the object if all edge functions are nonnegative for objects described by a series of clockwise vertices, or if all edge functions are negative for objects described by a series of counterclockwise vertices. Some
10 edge functions are effectively infinitesimally displaced from their corresponding edge, so that edges that are shared between adjacent objects assign points directly on the edge to exactly one of the objects.

15 The edge functions are evaluated at several points near the current stamp position. The sign bits of the edge functions at these points are combined to determine if the next position of the stamp should be one of the nearby positions, if the next position should be fetched from a previously stored context, or if all locations within the object have been visited. These sign bits are also combined to determine which, if any, of the nearby locations should be stored into their corresponding contexts.
20

In contrast to the prior art, which uses only four probe points at the vertices of the stamp rectangle, the present invention uses several new probe points near the stamp rectangle. These probes yield information that sometimes allows the stamp to avoid moving
25 immediately to a nearby unproductive position. In other cases, these new probe points yield information that allows the stamp to avoid restoring a saved stamp context that would be unproductive.

30 In some cases, the present invention uses the additional probe points to determine that certain moves would be unproductive.

In some cases, in order to avoid unproductive positions, the invention combines information from the additional probe points with a bit that indicates whether the stamp is in the first column of the object, or at some subsequent column.
35

In some cases, the invention uses information gleaned from the additional probes in order to invalidate positions previously saved in a stamp context. These positions were marked as unproductive at the time they were saved. Only after visiting subsequent positions can it be determined if such saved positions need not be visited, or if these saved positions must be restored and visited in order to reach other, possibly productive positions.

Finally, in some cases, visiting unproductive positions is unavoidable, as these positions provide a path to more distant stamp positions that will generate fragments. Very thin objects may, in fact, create a “stitching” effect, in which a few sample points are in the object, the next few are not, the next few are, etc.

In another aspect of the invention, the x and y coordinates of the probes and sample points are transposed, so that the invention may traverse the object by column stamplines rather than by row stamplines, without changing the movement logic.

In another aspect of the invention, the x and y coordinates of the starting vertex are adjusted before computing the starting stamp position, so that the invention may avoid an unproductive starting position.

In another aspect of the invention, the minimal bounding box of the object is slightly reduced in size to avoid visiting stamp positions that contain a portion of the object, are unproductive because the object does not extend far enough into the object to contain any sample points, but the object’s shadow is spanned by some segment that indicates the position is productive.

In another aspect of the invention, the preferred starting vertex is chosen so as to exploit the asymmetrical placement of probe points around the stamp.

In this invention, as in prior art, only the stamp positions immediately above, below, left, and right of the current position are considered to be nearby. This is not an inherent limitation of the invention, but rather an implementation issue. Allowing more positions (such as the four diagonally adjacent positions) to be considered nearby means that more logic, more gate delays, and possibly more saved contexts are involved in the decision of what position to move to next, and what nearby positions should be saved in stamp contexts. Adding more nearby positions slightly reduces the number of positions visited, at

the cost of a longer cycle time. This may make the invention *less* efficient, by requiring more overall time to visit all productive positions within an object.

BRIEF DESCRIPTION OF THE DRAWINGS

Embodiments of the invention will be described with reference to the accompanying drawings, in which:

Fig. 1 is a block diagram of a graphics processing system that can use the efficient polygon traversal according to the invention;

Fig. 2 is a diagram of a triangle with associated half-plane edges;

Fig. 3 is a diagram of a prior art fragment stamp with a minimal number of probes;

Fig. 4 is a diagram of a triangle which intersects all of the stamp's edge segments;

Fig. 5 is a diagram of a triangle and its shadow;

Fig. 6 is a diagram of a stamp edge segment that satisfies the first intersection test even though the segment is outside the object;

Fig. 7 is a diagram of a thin triangle that covers a few widely spaced non-contiguous sample points;

Fig. 8 is a diagram of the order in which the stamp traverses an exemplary triangle;

Fig. 9 is a diagram of the order in which an alternative embodiment traverses the same triangle as shown in Fig. 8;

Fig. 10 is a diagram of a fragment stamp augmented with several probes;

Fig. 11 is a diagram of the fragment stamp in Fig. 10, but with transposed probes;

Fig. 12 is a diagram of coincident probes in a one pixel by one pixel fragment stamp;

Fig. 13 is a diagram of a triangle with a sliver position above the stamp;

Fig. 14 is a diagram of a triangle with sliver positions above and below the stamp;

5 Fig. 15 is a diagram of a triangle with a special first-column sliver position above the stamp;

Fig. 16 is a diagram of a triangle that demonstrates why the special first-column sliver test should not occur in the last column stampline of an object;

10 Fig. 17A is a flow diagram for a fragment stamp traversal method according to one embodiment of the invention;

Fig. 17B is a flow diagram for a fragment stamp traversal method according to another embodiment of the invention;

15 Fig. 18 is a diagram of a minimal bounding box for a triangle;

Fig. 19 is a diagram of a graphics engine;

20 Fig. 20 is a diagram of a portion of the fragment generator of Fig. 19 that generates edge contexts;

Fig. 21 is a diagram of a portion of the fragment generator of Fig. 19 that generates channel contexts;

25 Fig. 22 depicts the positions for which edge function values are evaluated by the edge evaluators of the fragment generator;

Fig. 23 depicts an edge context data structure, which is generated by the fragment generator;

30 Fig. 24 depicts a channel context data structure, which is generated by the fragment generator;

Fig. 25 depicts a fragment stamp with four sample points per pixel for antialiasing;

35

Fig. 26 depicts a fragment stamp with four probe points arranged as a rectangle larger than the stamp rectangle;

Fig. 27 depicts a sample point that is shared between three triangles; and

Fig. 28 depicts a bounding box and a reduced bounding box for an exemplary triangle.

DESCRIPTION OF PREFERRED EMBODIMENTS

System Overview

Fig. 1 shows a computer system 100 embodying the principles of the invention. The system 100 can generate monochrome or multicolor 2-D and 3-D graphic images for rendering on a display device. In the computer system 100, a system chip set 104 provides an interface among a processing unit 102, a main memory 106, a graphics accelerator 108, and devices (not shown) on an I/O bus 110. The processing unit 102 is coupled to the system chip set 104 by the host bus 112 and includes a central processing unit (CPU) 118. The main memory 106 interfaces to the system chip set 104 by bus 114.

The graphics accelerator 108 is coupled to the system chip set 104 by a bus 116, to a graphics memory 122 by a bus 124, and to a display device 126 by a bus 127. The display device 126 includes a raster display monitor 128 for rendering color images on, for example, a display surface or screen 130. The invention can also be practiced with a monochrome monitor that displays gray-scale images, with a printer that prints black and white or color images, or with any other pixel-based output device such as a liquid-crystal or dot matrix displays.

The rendering surface 130, for example, a display screen, includes a 2-D array of data elements called pixels and produces an image 132 by illuminating a particular pattern of those pixels 134. Conventionally, the pixels have (x, y) Cartesian coordinates. The image 132, for example, can be 2-D alphanumeric characters or a 3-D scene filled with objects.

The graphics memory 122 includes storage elements for storing an encoded version of the graphical image 132. There is a direct correspondence between the storage elements and the pixels 134 on the display screen 130. The values stored in the storage elements for a

particular pixel, referred to as pixel data, control the intensity of the particular pixel 134 on the screen 130.

General Operation

During operation, the processing unit 102 can issue graphics commands requesting that a complex graphical object be rendered into an image 132. The processing unit 102 first tessellates the graphical object into primitive objects such as triangles, lines, or quadrilaterals, or into lists of such primitives. Each primitive directly or indirectly specifies a convex polygon of three or more sides. The chip set 104 sends graphics commands specifying such primitives to the graphics accelerator 108, which executes the commands, converting the primitive objects into fragments.

A fragment is the information associated with a 2-D polygon created by clipping a convex polygonal primitive of the image 132 to the boundaries of a pixel. Fragment information includes the x and y coordinates of the pixel; in this description, x coordinates increase from left to right, and y coordinates increase from top to bottom. Fragments also include *channel* information that is interpolated from values provided at the primitive's vertices, such as the *red*, *green*, and *blue* color values of the primitive object at that location, *alpha* transparency, *Z* depth value, texture coordinates, and the like.

The graphics accelerator 108 merges or replaces existing pixel data with data from the fragments, and loads the pixel data corresponding to the fragments into the appropriate storage elements of the graphics memory 122.

An important operation of the graphics accelerator 108 during graphics rendering is to determine which fragments are contained within a convex polygonal object. The graphics accelerator 108 initially positions a fragment stamp (preferably having a 2^m pixel wide by 2^n pixel high rectangle) so that the stamp rectangle contains one vertex of the object. Typically, the stamp is aligned to an x and y position that is a multiple of the stamp's width and height, respectively, while the vertices are specified to subpixel precision. The initial position of the stamp is computed by setting the appropriate number of lower bits of the starting vertex's (possibly adjusted) x and y coordinates to zero. Though the detailed methods below can start at any vertex on the edge of a minimal bounding box, for simplicity of description, most examples described herein starts at the left-most vertex of the object.

Edge functions are evaluated at several points on or near the stamp, yielding information about which nearby stamp positions probably contain portions of the object (i.e., fall at least partially within the boundary of the object). This information is used to determine nearby stamp positions to be visited immediately or sometime later. For the various embodiments of the invention described herein, the nearby stamp positions considered are the “Manhattan” stamp positions, which are directly left, right, up, and down from the current position. It will become apparent that the methods described herein can be extended to consider diagonally adjacent stamp positions or even nonadjacent positions, in order to visit even fewer nonproductive stamp positions that generate no fragments. However, the extra circuitry required for non-Manhattan movement may increase overall cycle time so much as to outweigh the small reduction in the number of moves used to traverse an object.

A nearby stamp position is *valid* if the traversal logic or process considers it a plausible candidate for visiting, and *invalid* if the position is not a candidate. Valid positions are those for which the stamp rectangle probably contains a portion of the object, and may have a sample point contained by the object. Valid positions are determined by approximating the mathematical intersection of a line segment between two of the probe points with the graphics object. Higher efficiency in fragment stamp movement is achieved when many unproductive positions are classified invalid, and hence not moved to.

More complex implementations of the graphics accelerator evaluate the edge functions at more points, and thus are able to classify some valid positions as *slivers*. A sliver position is a valid position which the traversal logic or process determines will nonetheless not generate any fragments, and determines that all stamp positions further on in the direction of the sliver position from the current position will also not generate any fragments. The embodiments described detect such slivers if the object lies completely to one side of all the sample points in the stamp. Movement to sliver positions can often, but not always, be avoided if a more promising valid position is available. Higher efficiency in fragment stamp movement is achieved when many sliver positions are avoided.

In some instances, visiting unproductive positions is unavoidable, as these positions provide a path to more distant stamp positions that will generate fragments. Fig. 7 shows such an example, where only three widely-separated fragments 710 marked with an X are contained in the triangle 700. In this case, it may be necessary to move through unproductive stamp positions to reach the fragments 710.

Movement of the stamp can be either directly to a nearby position (e.g., adjacent to the current stamp position), or by restoring a previously saved stamp context. The method also determines what nearby positions (if any) should be saved to the corresponding stamp contexts.

Example of Order of Traversal Using Bidirectional Movement in Stamplines

Two simple traversal algorithms that use extra probe point data are shown below. These are simplified, non-tiling versions of the more sophisticated traversal algorithms described in the above-referenced U.S. patent application. These simple algorithms initially position the fragment stamp near one vertex of the object, for example, the leftmost vertex. The stamp is initially positioned such that its rectangle is aligned to an (x, y) position that is a multiple of the stamp's width and height, and it encloses the starting vertex.

In the first traversal algorithm (also referred herein as a bidirectional movement algorithm), movement in each stampline first proceeds in one direction, then in the opposite direction. For later consistency of reference in computing the validity and sliver status of nearby positions, a stampline is considered to be (multiple) columns of pixels in this method. The column's width is equal to the stamp's width. This bi-directional movement algorithm could, naturally consider a stampline to be (multiple) rows of pixels.

This bidirectional movement algorithm uses three stamp contexts: the *current* context, as well as *backSave* and *overSave* saved contexts. A stamp context contains all the information needed by a graphics processor to place the stamp at a given position.

If the position above the starting position is valid, then that position is saved in the *backSave* stamp context.

The method moves to all valid stamp positions below the starting position, if any exists, then restores the *backSave* context and visits all valid positions above the starting position. When a context is restored, it is copied into the *current* context. Restoring a context also empties the restored context (or, equivalently, invalidates the restored context); a new position must be stored in the context before it can be restored again. If the *backSave* context is empty, that is, there was no valid position above the first position in the column stampline, this step is skipped.

As the stamp visits each position in the column stampline, also examined is the position in the column stampline immediately to the right of the current position. The first such valid position is saved in the *overSave* context.

5 When the stamp has finished visiting all positions on the current column stampline, the stamp then moves right to the *overSave* position in the next stampline. That is, the graphics accelerator restores the *overSave* context by copying it into the *current* context and invalidating the *overSave* context. The graphics accelerator repeats the process of storing into *backSave* the position above the first position in the new stampline if that position is
10 valid, visiting all the valid positions below, restoring the *backSave* context (if valid) and visiting all the valid positions above, and then moving to the next stampline to the right by restoring the *overSave* context. When the stamp has no positions farther to the right to visit (i.e., when it attempts to restore *overSave*, but that context is empty), the graphics accelerator is finished traversing the object.

15 In the bi-directional movement algorithm, whenever a valid *over* or *overSave* position exists, the algorithm never moves to a *forward*, *back*, or *backSave* position that was determined to be a sliver. By definition, sliver positions can never lead to a valid position within the current stampline, and so the algorithm moves over to the next stampline as soon
20 as possible. Such sliver positions are visited only if there is no valid position in the next stampline over. In this case, sliver positions *must* be visited, as they may lead to a position in the current stampline for which there is a valid position in the next stampline over.

Bypassing Saved Contexts

25 In this and some embodiments of the invention discussed below, bypassing is used to avoid the time required to save a nearby stamp position in one cycle, and then immediately load that saved context in the next cycle. For example, if the first stamp position in a stampline has no valid position to visit below, then for the next cycle the stamp immediately proceeds
30 to the valid position above, rather than taking one cycle to save the above position in *backSave*, and another cycle to restore it from *backSave* into the *current* context.

Similarly, if the position to the right is valid and *overSave* is empty when all locations in a stampline have been visited, then the graphics accelerator immediately moves the stamp
35 right to the next stampline rather than saving the position in *overSave* and then restoring that position on the next cycle.

As can be seen in the detailed descriptions, bypassing increases the complexity of the traversal logic and process. For clarity, the summaries of each method always refer to saving and restoring a context, even when the implementation actually bypasses the saved context for efficiency.

5

Fig. 8 depicts the order in which the fragment stamp moves when traversing a triangle 800. For simplicity, the fragment stamp is a single pixel wide and high. It should be apparent that arbitrary stamp sizes can be used. A pixel is considered to be inside the triangle if the upper left corner of its square is inside the triangle. This positioning of the sample point at the upper left corner makes implementing the computation of the edge functions simpler, and is an arbitrary choice. Each pixel inside the triangle has been labeled with a number showing the order in which pixels are visited.

10

Each *back* position above the first position on each stampline that was saved into *backSave* and then later restored is enclosed in a solid diamond. Each *back* position that was immediately bypassed directly into the *current* context is enclosed in a dashed diamond. Each *over* position to the right of the stampline that was saved into *backSave* and then later restored, or was immediately bypassed, is enclosed in a solid or dashed circle, respectively.

15

Note that the stamp moves from position 36 to position 37, which is a non-productive position. This non-productive move is needed to get from position 36 to position 38, which is productive. Note further that without the improvements enabled by additional probe points, the stamp would visit *every* pixel square which is fully or partially covered by the object, even if the object does not include the upper left corner of the pixel.

20

25

Example of Order of Traversal Using Unidirectional Movement in Stamlines

30

35

In an alternate embodiment of the invention, the order in which stamp positions are visited is altered by giving precedence to moving in the *over* (right or left) direction before moving in the *forward* or *back* direction. This different traversal order moves the stamp across each stampline in the same direction, rather than first in one direction and then the opposite direction. This ordering offers no advantages over the above embodiment for the purposes of this disclosure, and in fact reduces efficiency by more frequently moving to *over* positions that are not productive. (Even for very small triangles, where this effect is largest, the efficiency loss is under 1%.) In exchange, this traversal order offers substantial

advantages in reducing the number of contexts and decision-making complexity required when tiling constraints are added, as described in the aforementioned patent application.

This method again uses three stamp contexts: the *current* context, as well as *backSave* and *forwardSave* saved contexts. In this alternate embodiment, again suppose that the algorithm chooses the leftmost vertex of the object. Rather than generating stamplines that are (multiple) columns of pixels, the present method generates stamplines that are (multiple) rows of pixels.

The method first generates all fragments in the row stampline to the right of the starting position. As it visits each position in this row, the fragment stamps also looks for valid positions in the row stamplines above and below, and saves the first of each in *backSave* and *forwardSave*, respectively.

When the stamp has visited all valid positions in the row stampline, it then starts at the *forwardSave* position in the row below, by restoring *forwardSave* into the current stamp position. The stamp then visits all valid positions in that row, while looking for the first valid position farther below to store into *forwardSave*. It continues to restore *forwardSave* and traverse each row stampline until *forwardSave* is empty when an attempt is made to restore it. At this point, the stamp has visited all positions in the object that are in or below the starting stampline.

The stamp then moves to the position that was saved above the first stampline, by restoring the *backSave* context into the current stamp position. It visits all valid positions in that row stampline, saving a valid *backSave* position if it can find one, restoring *backSave* and generating the next row above, etc. When there is no valid position stored in *backSave* to restore, then all valid positions within the object have been visited, and object traversal is complete.

This algorithm also attempts to avoid sliver positions, but the different traversal order requires somewhat different rules. Again, whenever a valid *over* position exists, the algorithm never moves to a *forward*, *forwardSave*, *back*, or *backSave* position that is adjacent to the current stampline and which was determined to be a sliver. By definition, such moves can never lead to a valid position in the current column—they can only lead to a valid position in the next column over. But such a valid position will also be found if the

algorithm moves to the *over* position. These sliver positions are visited only if *over* is not valid. If *over* is a sliver, and a valid non-sliver *forward*, *forwardSave*, *back*, or *backSave* position exists, these take precedence over the *over* sliver. Again, by definition there will be no more productive positions in the current stampline, and so there is no point continuing in the *over* direction. Also, a valid non-sliver *forward* position should overwrite a sliver *forwardSave* position, and the same for *back* and *backSave*.

The behavior of this algorithm is shown for the exemplary triangle 900 in Fig. 9. Again, each pixel has been labeled with a number showing the order in which it was visited. In this example the *forwardSave* positions are shown in solid and dashed hexagons, and the *backSave* positions are shown in solid and dashed diamonds, where dashes indicate that a context is bypassed.

Implementation Details

The principle steps for the traversal methods according to the invention are shown in Fig. 17A. The steps are implemented by circuits and software of the graphics accelerator 108 of Fig. 1, which are shown in more detail in Figs. 20 and 21.

Additional Probe Locations

Avoiding unproductive stamp positions requires evaluating the edge functions at more locations than the four locations of prior art. In particular, according to the present invention, unproductive stamp positions are avoided by evaluating edge functions at probe points outside the stamp's rectangle. In a preferred embodiment, the stamp's rectangle is defined to be a rectangle of *stampWidth* pixels by *stampHeight* pixels, where all fragments that are within both the stamp and the object to be rendered are computed simultaneously. In the preferred embodiment, the possible locations where the fragment stamp can move to are contiguous but non-overlapping. Thus, in the preferred embodiment, probe points outside the stamp's rectangle when the stamp is at one stamp location will be inside the stamp's rectangle only when the stamp is at an adjacent stamp position.

A simple scheme might evaluate probes that still form a rectangle, albeit larger than the stamp's rectangle. Fig. 26 shows such a scheme for a stamp rectangle 2600. The prior art placement of probe points shown in Fig. 3 has been extended upward and leftward. *RT* has been moved upward one pixel, and *LL* has been moved leftward one pixel. A new probe

point *UPPER_LEFT* (*UL*) has been added in the upper left corner. The computation of valid positions is similar to prior art, but the segment (*UL*, *UR*) is tested instead of (*ORIGIN*, *UR*), and the segment (*LB*, *UL*) is tested instead of (*LB*, *ORIGIN*).

Such a scheme does avoid some unproductive positions. For example, if an object extends less than one pixel above the stamp, as does triangle 2610, this larger probe rectangle in some cases prevents the stamp from moving up to the unproductive position above stamp position 2600. However, there are still many situations in which such an augmented stamp will move to unproductive positions. Such positions cannot be avoided by pushing the probe points out even farther from the stamp rectangle. Probe points cannot be moved so far that the segments being tested will include sample points from nearby stamp positions, otherwise possibly productive positions will not be considered valid positions to which to move.

Instead, many more unproductive positions can be avoided by adding several probe points, carefully positioned around the stamp rectangle to extract a large amount of information about nearby positions. Fig. 10 illustrates a 4 pixels wide by 2 pixels high fragment stamp rectangle 1000 with additional probes that surround the stamp, where *SU* (described momentarily) is one pixel. The probes and their position relative to the upper left corner of the stamp rectangle are shown in the table below. *SU* is the smallest unit in the grid that sample points lie on. This grid is usually of equal or coarser grain than the grid in which vertex coordinates are specified. For aliased drawing, a pixel is either in the object or not in the object, and *SU* is 1 pixel. For supersampled antialiased drawing, a pixel may be partially covered by the object, and this coverage is represented by a bit string of several sample points within each pixel, where each sample point is either in or not in the object. In this case, the sample points are positioned upon a subpixel grid. If sixteen sparse sample points are used, this grid might be 1/16 of a pixel, and so *SU* would also be 1/16 of a pixel. The probe points are described in the table below.

<u>PROBE POINT</u>	<u>X OFFSET</u>	<u>Y OFFSET</u>
<i>ORIGIN</i>	0	0
<i>LEFT_TOP</i> (<i>LT</i>)	$-SU$	0
<i>UPPER_LEFT</i> (<i>UL</i>)	0	$-SU$
<i>UPPER_RIGHT</i> (<i>UR</i>)	$StampWidth - SU$	$-SU$

<i>RIGHT_TOP (RT)</i>	<i>StampWidth</i>	0
<i>LEFT_BOTTOM (LB)</i>	$-SU$	<i>StampHeight</i>
<i>LOWER_LEFT (LL)</i>	0	<i>StampHeight</i>
<i>LOWER_RIGHT (LR)</i>	$StampWidth - SU$	<i>StampHeight</i>
<i>RIGHT_BOTTOM (RB)</i>	<i>StampWidth</i>	<i>StampHeight</i>

This placement of probe points assumes that antialiased sample points are placed with some degree of regularity, in that each row and column of the grid contains one sample point. If a less regular pattern of sample points is used, the position of the probe points may need to be adjusted slightly. The segments between probe points that are tested in the methods described below should extend as far out from the stamp as possible, without extending so far as to allow an object to contain a sample point but not intersect the appropriate segments. If, for example, an antialiased arrangement did not place any sample points on the bottommost row of the grid, the *UL* and *UR* probe points y offset from the origin could be increased to $-2SU$.

The alternative implementation (using unidirectional movement in stamplines) can be improved slightly by using two more probe points, which are described in the table below:

<u>PROBE POINT</u>	<u>X OFFSET</u>	<u>Y OFFSET</u>
<i>LEFT_MIDDLE</i>	$-SU$	$StampHeight - SU$
<i>RIGHT_MIDDLE</i>	<i>StampWidth</i>	$StampHeight - SU$

Note that only one of these probes is active for the traversal of a given object. Thus, these two additional probe points can be implemented by multiplexing the appropriate values into a single adder.

In some cases, these additional probe points do not provide sufficient information to avoid some moves to unproductive positions. Fig. 4 provides an example. Although the triangle 420 intersects the right edge (*RT*, *RB*) of stamp 410, the triangle slips between the two left-most sample points of the stamp position to the right, that is, the grid point *RT* and the grid point immediately below *RT*. Correctly avoiding moving to the position to the right is much more involved than avoiding the position to the left of the stamp. For example, a slightly different triangle might slip through the same two left-most sample points of the stamp position to the right, but nonetheless enclose a sample point even farther to the right.

Detecting such cases requires adding many more probe points. However, while probe points are relatively cheap in implementation cost, they are not free. Of more concern is the minimum cycle time required to move the stamp. The more probe points used, the more information has to be combined before moving the stamp, which in turn increases gate delays. Because stamp movement cannot be pipelined, it may well prove one of the critical delay paths in a design, and so this increased gate delay may translate to a longer cycle time. In this case, adding more probe points may make the stamp more “efficient” measured in the number of cycles it takes to generate all fragments within an object, but less efficient measured in the time it takes to generate the fragments due to the increased cycle time.

Transposed Probe Points

The following discussion will be in terms of up being *back*, down being *forward*, and left or right (depending if the starting position was the right-most or left-most vertex) being *over*. The stamp algorithms also work for left being *back*, right being *forward*, and up or down being *over*, by transposing the probe points. Here, transpose means computing the probe offsets by swapping the roles of *x* and *y* offset columns, *and* by swapping the roles of *StampWidth* and *StampHeight* in the swapped columns, as shown for the stamp 1100 in Fig. 11 and in the following table. (The probe names no longer make much sense when they are transposed.) This transposition forces the movement algorithm to reverse the roles of rows and columns without any changes to the algorithm, and thus avoids additional gate delays in the movement logic. The transposed probe points are described in the table below.

<u>TRANSPOSED PROBE POINT</u>	<u>X OFFSET</u>	<u>Y OFFSET</u>
<i>ORIGIN</i>	0	0
<i>LEFT_TOP (LT)</i>	0	– <i>SU</i>
<i>UPPER_LEFT (UL)</i>	– <i>SU</i>	0
<i>UPPER_RIGHT (UR)</i>	– <i>SU</i>	<i>StampHeight</i> – <i>SU</i>
<i>RIGHT_TOP (RT)</i>	0	<i>StampHeight</i>
<i>LEFT_BOTTOM (LB)</i>	<i>StampWidth</i>	– <i>SU</i>
<i>LOWER_LEFT (LL)</i>	<i>StampWidth</i>	0
<i>LOWER_RIGHT (LR)</i>	<i>StampWidth</i>	<i>StampHeight</i> – <i>SU</i>

<i>RIGHT_BOTTOM (RB)</i>	<i>StampWidth</i>	<i>StampHeight</i>
<i>LEFT_MIDDLE</i>	<i>StampWidth – SU</i>	<i>– SU</i>
<i>RIGHT_MIDDLE</i>	<i>StampWidth – SU</i>	<i>StampHeight</i>

5 Probe point transposition can be accomplished in one of two ways, depending upon the number of sample points vs. the number of probe points, and cycle time constraints. The probe points can be moved by multiplexing different offsets into the adders that compute the edge functions at each probe point. This adds multiplexors into the path required to compute the edge functions at the probe points, which in turn increases the time required to
10 move the stamp.

15 Alternatively, the roles of *x* and *y* can be swapped during the setup of the edge evaluators, so that no additional multiplexing is required to transpose the probe points. However, note that sample points should not be transposed. This scheme in general requires multiplexing different values into the sample point adders, depending on if probes are transposed or not. As a special case, if the stamp is a square and the set of sample points is symmetrical around the (*x*, *y*) diagonal, the mask of sample points bits output from the sample point computation logic can be multiplexed. In either case, multiplexing the inputs or outputs of the sample point computation logic probably does not increase the cycle time of the stamp, as these
20 values are not subject to further computation by the stamp movement logic. (The exception is the *ORIGIN*, which fortunately needs no multiplexing nor adders to compute.)

25 Alternatively, the stamp can be surrounded by additional probe points, which evaluate the union of the set of non-transposed probe points and the set of transposed probe points. In this case, movement algorithms similar to those described here can be derived by one skilled in the art, in which one subset of the union set of probes is used to traverse row stamplines, and another subset is used to traverse column stamplines. Again, though, this technique requires additional multiplexors in the stamp movement decision making logic, and so may increase the time required to move the stamp.

30

Coincident Probe Points

35 If the stamp is one pixel wide (or one pixel high using the transposed probe points), and the sampling grid unit *SU* is also one pixel, some of the probe points become coincident (that is, occupy the same location). In particular, *UPPER_LEFT* and *UPPER_RIGHT* become coincident, and *LOWER_LEFT* and *LOWER_RIGHT* become coincident. Fig. 12 shows a

one pixel wide by one pixel stamp 1200 using the non-transposed probes (that is, up is *back*, down is *forward*, and left or right is *over*). Both movement algorithms described below work with coincident probes. If the stamp is implemented as a constant 1x1 pixel, the algorithms below can be slightly simplified by merging these coincident probe points.

Alternatively, the stamp may be implemented with variable size stamps (for example, 4x1 pixels when texture mapping, 4x2 pixels when not) without special logic to handle coincident probe probes.

Some Advantages of Asymmetrical Placement of Sample and Probe Points

The fragment stamps shown in Figs. 10, 11, and 12 have a smallest sampling unit SU of 1, and place the sample points and the probe points at what is apparently the “upper left corner” of pixels. This placement matches semantics of the X11 Window System, in which integer pixel coordinates are considered to be the “center” of a pixel. That is, X11 defines the pixel at integer coordinates (x, y) to “own” the half-open square with upper left corner $(x-1/2, y-1/2)$ and lower right corner $(x+1/2, y+1/2)$. However, this placement does not match the semantics of OpenGL, in which half-pixel coordinates are considered to be the “center” of the pixel, and so the pixel at integer coordinates (x, y) “owns” the half-open square with upper left corner (x, y) and lower right corner $(x+1, y+1)$. OpenGL semantics are easily accommodated with this arrangement of sample points by adding $1/2$ to each object vertex.

Similarly, if SU is smaller than 1, the multiple sample points per pixel will not be placed symmetrically around the “center” of each pixel, but will be placed asymmetrically, biased toward the upper left corner of each pixel. For example, if SU is $1/4$, then four samples points might be placed in each pixel as shown in Fig. 25. In general, OpenGL semantics are accommodated with such placements by adding $SU/2$ to each object vertex.

Mathematically, the placement of sample points (and thus probes) within pixels is arbitrary. One skilled in the art can easily adapt the invention to a configuration of sample points that are placed symmetrically around the “center” of the pixel, for example by adding $SU/2$ to the x and y coordinate of each sample point and probe point.

However, asymmetrical placement offers implementation and efficiency advantages. In terms of the implementation of sample points and probe points, fewer inputs to the adders computing the edge functions at each point are required, and these adders need not negate any of their inputs. Further, as discussed below in conjunction with Fig. 17A, steps 1702,

1703 and 1705, this asymmetry can be exploited to reduce the number of cycles required to completely traverse and object.

Method using Bidirectional Movement in Stamplines

As mentioned above, an embodiment of the present invention uses bi-directional stamp movement and has three stamp contexts: the *current* context, and the saved contexts *backSave* and *overSave*.

In one implementation, the saved context *backSave* has associated with it two bits, *backSaveValid*, and *backSaveSliver*. The bit *backSaveValid* is true when a valid stamp position is stored in the context, otherwise the context is empty. The bit *backSaveSliver* is true when *backSaveValid* is true and the position saved is an unproductive “sliver” position that may not need to be visited or saved, as discussed more fully below in the descriptions of steps 1708 through 1710. If *backSaveSliver* is false, it is unknown if the position is productive or non-productive.

The saved context *overSave* has associated with it two bits, *overSaveValid* and *overSaveProductive*. The bit *overSaveValid* is true when a valid stamp position is stored in the context, otherwise the context is empty. The bit *overSaveProductive* is true when *overSaveValid* is true and the position saved is known to be a productive position that must be visited, as discussed more fully below in the descriptions of steps 1708 through 1710. If *overSaveProductive* is false, it is unknown if the position is productive or non-productive.

The saved contexts represent the following positions:

1. *backSave*: the position above the first stamp position in the column stampline.
2. *overSave*: the first best position found to the right (if started at left-most vertex), or left (if started at right-most vertex) of the current stampline. “First best” means that a position known to be productive can replace a saved position that is not known to be productive.

Step 1701: Determine the minimal bounding box for object.

Fig. 17A, step 1701 determines the minimal rectangular bounding box (*bbox*) that encloses the object and whose sides are parallel to the *x* and *y* axis. For example, Fig. 18 shows a triangle 1800 drawn with solid lines, and its minimal bounding box 1810 drawn with dashed lines. In this embodiment, we allow both three-sided objects like triangles, and four-sided objects like quadrilaterals and rectangular lines, so we allow up to four vertices (x_0, y_0), (x_1, y_1), (x_2, y_2), and (x_3, y_3). For simplicity of this description, assume that for triangles, the (x_2, y_2) vertex is copied into the (x_3, y_3) vertex. C++ code for Fig. 17A, step 1701 is:

```
bbox.xmin = min(x0, x1, x2, x3);
bbox.xmax = max(x0, x1, x2, x3);
bbox.ymin = min(y0, y1, y2, y3);
bbox.ymax = max(y0, y1, y2, y3);
```

It is noted here that the listing of C++ code in this document does not mean that the graphics accelerator is implemented using a general purpose processor that executes C++ code. Rather, the C++ code appearing herein (including Appendices A to H and elsewhere in this document) specifies the logical operations of certain portions of the circuitry of the graphics accelerator.

Step 1702: Select a starting vertex on a side of the bounding box.

In Fig. 17A, step 1702 selects a *starting* vertex (x_{start}, y_{start}) that is on an edge segment of the bounding box *bbox*, where *start* is in the range [0, 3]. Such a vertex is called a *single-extreme* vertex, as this vertex is at an extreme *x* or *y* position of the bounding box. For triangle 1800, all three vertices 1801, 1802, and 1803 are single-extreme vertices.

In this embodiment, if the desired traversal order specifies column stamplines, the starting vertex must be on the left or right side, that is, (x_{start}, y_{start}) must satisfy the condition:

$$x_{start} == bbox.xmin \parallel x_{start} == bbox.xmax$$

In this embodiment, non-transposed probes are used.

In another embodiment, if a traversal order specifies row stamplines, the starting vertex must be on the top or bottom of the bounding box, that is, (x_{start}, y_{start}) must satisfy the condition:

$$y_{start} == bbox.ymin \parallel y_{start} == bbox.ymax$$

In that embodiment, transposed probes are used.

5 In Fig. 17A, step 1705, described below, a slightly shrunk bounding box is aligned to the stamp size. If the sample points and probe points are asymmetrically placed (for example, shifted toward the upper left as in the preferred embodiment), the bounding box size will be reduced asymmetrically. On average, the number of cycles required to traverse an object is minimized if the starting vertex is chosen to lie on the bounding box side that is opposite the most reduced side of step 1705. In the preferred embodiment, the top and left sides of the bounding box are most reduced in step 1705, so when possible, the bottom or rightmost vertex is preferentially chosen as the starting vertex. In some cases, the type of object forces the choice of starting vertex, and so this optimization is disabled. For example, when painting a line with a dash pattern, it is easiest to paint from the beginning of the line to the end of the line.

There are implementation advantages, such as combining the *backSave* and *forwardSave* contexts, to starting at the *corner* of the bounding box. That is, (x_{start}, y_{start}) satisfies the condition:

$$(x_{start} == bbox.xmin \parallel x_{start} == bbox.xmax) \&\& (y_{start} == bbox.ymin \parallel y_{start} == bbox.ymax)$$

Such a vertex is called a *double-extreme vertex*, as it is at both an extreme *x* position and an extreme *y* position of the bounding box. For triangle 1800, the vertex 1801 is a double-extreme vertex. It is always possible to find a double-extreme vertex for triangles, thin OpenGL lines, Microsoft Windows lines, thin X11 lines, and wide OpenGL aliased lines. However, it is not possible to find a double-extreme vertex for X11 wide lines, nor for OpenGL antialiased lines. These objects may still be rendered using fewer contexts by splitting them into two portions, each of which has a double-extreme vertex. An algorithm that uses fewer states by starting at a double-extreme vertex, will not be described. But such an algorithm can be easily derived by merging the *back* and *forward* sparse contexts, as well as the *backSave* and *forwardSave* contexts, and by setting *overProductive* true at every position. Note that while always choosing a double-extreme vertex reduces implementation complexity, it decreases efficiency by requiring the stamp to start

sometimes at the upper leftmost vertex, and by eliminating the effectiveness of the *overProductive* bit.

Step 1703: Align the starting position to the stamp size

In general, the starting vertex (x_{start}, y_{start}) is specified with subpixel accuracy, e.g., ($19^{15/16}, 34^{2/16}$). The origin of the stamp, though, typically is preferably aligned to an (x, y) position commensurate with the stamp rectangle's dimensions. For example, if the stamp rectangle is four pixels wide by two pixels high, then the starting position is preferably aligned so that the x position is a multiple of four pixels, and the y position is a multiple of two pixels.

A special tie-breaking rule must be applied if a directed edge, or a vertex, falls exactly on a sample point (that is, the edge function or both edge functions evaluate to 0 at the sample point). Fig. 27 shows an example in which three triangles 2710, 2720, and 2730 all share a vertex at sample point 2740. The tie-breaking rule ensures that exactly one of these three triangles will include the sample point 2740. Which triangle the tie-breaking rule assigns the sample point to is arbitrary. A typical tie-breaking rule is to include a sample point that does not lie on a right edge of a triangle; if the sample point is on a horizontal edge it is included if the horizontal edge is the top of the triangle and if the sample point is not on a right edge of the triangle. This rule is hereafter referred to as "left top." There are seven other similar rules: "top left," "left bottom," "bottom left," "right bottom" etc. With the "left top" tie-breaking rule, for example, sample point 2740 of Fig. 27 is included in triangle 2740, but is not included in triangles 2710 and 2730 because it is on a right edge of each of these triangles. Without loss of generality, the optimizations described immediately below and in conjunction with Step 1705 apply to the "top left" and "left top" tie-breaking rules for stamps which contain sample points on the left and top edges of the stamp. As should be obvious to one versed in the arts, similar optimizations apply to the other six tie-breaking rules. Similarly, similar optimizations apply to other arrangements of sample points.

With the "top left" and "left top" tie-breaking rules, there is no need to visit stamp positions in which the right-most vertex is exactly on the left edge of the stamp rectangle, or in which the bottom-most vertex is exactly on the top edge of the stamp rectangle. For example, in Fig. 27, there is no need to visit stamp position 2700 when rendering triangle 2710, as the tie-breaking rules guarantee that the rightmost vertex of triangle 2710 cannot include a sample point of the stamp at that position.

If a right-most starting vertex lies on a stamp rectangle edge, and/or a bottom-most starting vertex lies on a stamp rectangle edge, the starting coordinates can be adjusted by the subpixel precision *epsilon* so that the stamp will start in the position to the left and/or above the starting vertex. The subpixel precision *epsilon* is the granularity with which the graphics accelerator internally represents vertex coordinates used to generate fragments, and is usually equal to or finer than the antialiasing sample grid spacing *SU*, described above. The subpixel precision *epsilon* may be much coarser than the granularity with which the application may specify vertex coordinates, especially if the graphics accelerator accepts such coordinates in a floating point representation.

This alignment of the starting position to the stamp size is performed in Fig. 17A, step 1703:

$$\begin{aligned}x_{adj} &= x_{start} - ((x_{start} == bbox.xmax) ? epsilon : 0); \\y_{adj} &= y_{start} - ((y_{start} == bbox.ymax) ? epsilon : 0); \\x_{alignedStart} &= x_{adj} - (x_{adj} \bmod stampWidth); \\y_{alignedStart} &= y_{adj} - (y_{adj} \bmod stampHeight);\end{aligned}$$

In the preferred embodiments, *stampWidth* and *stampHeight* are both powers of two, and so the alignment can be performed more efficiently as a masking operation:

$$\begin{aligned}x_{alignedStart} &= x_{adj} \& \sim(stampWidth - 1); \\y_{alignedStart} &= y_{adj} \& \sim(stampHeight - 1);\end{aligned}$$

If the x_{adj} and y_{adj} coordinates above are fixed-point values with one or more bits of subpixel precision, they must first be shifted right by the number of subpixel bits before being masked.

Note that if *epsilon* is sufficiently small (e.g., smaller than 1/16), the starting vertex will rarely trigger this condition, and this adjustment may not be worth the implementation cost.

There is no simple way to similarly adjust the starting position for a left-most or top-most starting vertex that is to the right of the right-most sample point, or below the bottom-most sample point of the stamp. Instead, the invention avoids such situation by preferentially choosing a right-most or bottom-most starting vertex in Fig. 17A, step 1702, described above.

Step 1704: Initialize the edge evaluators

In Fig. 17A, step 1704, the edge evaluators are initialized for the aligned starting position. This involves computing the increments A and B for each edge function $E(x, y) = Ax + By + C$, and computing the value each edge function at the position $(x_{alignedStart}, y_{alignedStart})$. The setup for the edge evaluators is described by Pineda in the above mentioned article.

Step 1705: Determine a bounding box aligned to the stamp size

Fig. 17A, step 1705 creates *extent*, a version of the minimal bounding box aligned to the stamp size. This simplifies the implementation of the second intersection test, which ensures that a valid stamp position is not outside the bounding box of the object. The minimal bounding box is slightly reduced in size before aligning its edges, so as to avoid visiting stamp positions in which the object protrudes so slightly that it cannot include any sample points, but whose shadow spans a relevant segment between probes.

As mentioned above in the description of Fig. 17A, step 1703, there is no need to visit stamp positions in which the right-most vertex is exactly on the left edge of the stamp rectangle, or in which the bottom-most vertex is exactly on the top edge of the stamp rectangle. Step 1705 adjusts the maximum x and y values of the bounding box before alignment so that any such stamp position will be considered outside the object

Similarly, there is no need to visit stamp positions in which the left-most vertex is to the right of the right-most sample point in the stamp ($stampWidth - SU$ in the preferred embodiment), or in which the top-most vertex is below the bottom-most sample point in the stamp ($stampHeight - SU$). Step 1705 adjusts the minimum x and y values of the bounding box before alignment so that any such stamp position will be considered outside the object. The following C++ code shows how the reduced bounding box is computed:

```
reduced.xmin = bbox.xmin + SU - epsilon;  
reduced.xmax = bbox.xmax - epsilon;  
reduced.ymin = bbox.ymin + SU - epsilon;  
reduced.ymax = bbox.ymax - epsilon;
```

Fig. 28 shows how the bounding box 2860 and the reduced bounding box 2870 are computed for a triangle 2850. In this example, the sampling unit SU is 1 pixel, while the subpixel precision $epsilon$ is 1/4 pixel. The bounding box 2860 is the smallest rectangle

than contains the triangle 2850. Since the right-most vertex of triangle 2850 falls on the right edge of the stamp rectangle at position 2810, the right edge of the bounding box 2860 is moved to the left by 1/4 pixel (*epsilon*) to create the right edge of the reduced bounding box 2870. The bottom-most vertex of triangle 2850 does not fall on the edge of a stamp rectangle, and so the bottom edge of reduced bounding box 2870 is at the same vertical position as the bottom edge of bounding box 2860. The top and left edges of bounding box 2860 are moved down and right, respectively, by 3/4 pixels (*SU - epsilon*) to create the top and left edges of reduced bounding box 2870. Note that since triangle 2850 does not project far enough up to contain any sample points at stamp position 2800, and does not project far enough left to contain any sample points at stamp positions 2820 or 2830, this adjustment shrinks the reduced bounding box 2870 so that these positions will not be visited.

The stamp-aligned bounding box *extent* can be computed from *reduced* as follows:

```

extent.xmin    = reduced.xmin - (reduced.xmin mod stampWidth);
extent.xmax    = reduced.xmax - (reduced.xmax mod stampWidth);
extent.ymin    = reduced.ymin - (reduced.ymin mod stampHeight);
extent.ymax    = reduced.ymax - (reduced.ymax mod stampHeight);

```

Again, since *stampWidth* and *stampHeight* are powers of two in the preferred implementation, the alignment may be implemented using masking operations rather than modulo operations, similar to those in Fig. 17A, step 1703.

In an actual implementation, the additions and subtractions of *stampWidth* and *stampHeight* mentioned in the description of step 1707 below are folded into the computation of the *reduced* bounding box.

Step 1706: Initialize the starting bookkeeping state of the invention

Fig. 17A, step 1706 initializes the values of all states used while traversing the object. The following are initial bookkeeping state values used herein, and the C++ code to initialize them:

```

if (!transposedProbes && x_start == bbox.xmin
    || transposedProbes && y_start == bbox.ymin)
    dirOver = POSITIVE
else

```

```

    dirOver = NEGATIVE;
    firstColumnInObject = true;           // Haven't made an over move yet?
    firstPositionInLine = true;           // First stamp position in this stampline
    dirForwardBack == POSITIVE // Moving forward or back in this stampline?

```

5 Initially, all saved contexts are invalid:

```

    overSaveValid = false;
    overSaveProductive = false;
    backSaveValid = false;
    backSaveSliver = false;

```

10 Note, steps 1707 through 1711, described in the following sections, are repeated until no further stamp moves are possible in step 1709. Thereafter, the process is repeated for another object.

15 Step 1707: Evaluate edge functions at each probe
and compute valid, sliver, and productive bits

For stamp movement purposes there are three additional “sparse” stamp contexts. The sparse stamp contexts contain much less information than the full contexts described above. These sparse contexts do not include all the information associated with the interpolated values of colors, Z depth, transparency, and so on, and instead only include the edge function values for these stamp positions. Furthermore, the edge function values for these positions are not stored in flip-flop or latch circuits of the graphic accelerator 108, but are determined anew each cycle by combinational logic. These sparse contexts are (with non-transposed probes):

- 25 1. *forward*: the stamp position immediately below the current position
2. *back*: the stamp position immediately above the current position
- 30 3. *over*: the stamp position immediately right of the current position if *dirOver* is POSITIVE, else the position immediately to the left.

At each stamp position, these three adjacent positions are examined to determine if they are *valid* (that is, should possibly be moved to). In addition, the *forward* and *back* positions are
35 examined to determine if they are *slivers*. Sliver positions are unproductive, and generate

no fragments, but may need to be visited in order to get to more distant productive positions. And, the *over* position is examined to determine if it is known to be *productive*.

Note that for the most part, *valid*, *sliver*, and *productive* bits are hints—*valid* may be falsely positive, and *sliver* and *productive* bits may be falsely negative. That is, a valid *forward* or *back* position that is not marked as a *sliver* may not even contain a portion of the graphics object, or may satisfy sliver semantics. A valid *over* position that is not determined to be productive may or may not actually be productive. In general, any feasible implementation has to settle for detecting a reasonable subset of the positions as *valid*, *sliver*, and *productive*.

Since several intersection tests may be used to set the *valid*, *sliver*, and *productive* indicators for the sparse stamp contexts, in an actual implementation it is convenient for the procedure *Intersects* to perform only the first portion of the intersection test. That is, the *Intersects* procedure returns true if the specified segment between probe points either intersects the object, or spans its shadow.

The second portion of the intersection test, that is, the test for the stamp position being inside the aligned bounding box *extent*, is performed separately for each of the nearby positions. The *valid*, *sliver*, and *productive* bits are then logically ANDed with the appropriate results of the bounding box test at convenient points in their computation. To compute the bounding box test, four values are first computed:

```
rightInsideObject = current.x + stampWidth <= extent.xmax;  
leftInsideObject  = current.x - stampWidth >= extent.xmin;  
downInsideObject  = current.y + stampHeight <= extent.ymax;  
upInsideObject    = current.y - stampHeight >= extent.ymin;
```

(In an actual implementation, the addition and subtraction of *stampWidth* and *stampHeight* are folded into the computations of the *reduced* bounding box described above in step 1705.) These four values are then mapped into the *forward*, *back*, and *over* contexts, depending on if probes are transposed or not:

```
if (!transposedProbes) {  
    forwardInsideObject = downInsideObject;  
    backInsideObject    = upInsideObject;  
    overInsideObject    =  
        (dirOver == POSITIVE ? rightInsideObject : leftInsideObject);  
} else {
```

```

    forwardInsideObject = rightInsideObject;
    backInsideObject = leftInsideObject;
    overInsideObject =
        (dirOver == POSITIVE ? downInsideObject : upInsideObject);
}

```

5

Determining *backValid* and *backSliver*

To test if the stamp can move up (e.g., to a position above the current position), the invention does not test if the object intersects the segment (*ORIGIN*, *RT*) as does prior art. The present invention instead tests for intersection with the smaller and higher segment (*UL*, *UR*), as well as one or both of the small diagonal segments (*LT*, *UL*) and (*UR*, *RT*).

10

If the object intersects (*UL*, *UR*), *backValid* is set true, and *backSliver* is set false. In this case, the object extends far enough upward to potentially include sample points along the segment (*UL*, *UR*). Even if the object does not contain *UL* or *UR*, or any other sample point along that segment (that is, the object slips between two adjacent sample points), it may nonetheless contain sample points higher up.

15

If the stamp is moving *over* from left to right, and the object does not intersect (*UL*, *UR*), but does intersect (*UR*, *RT*), the *back* position is marked as a *sliver*. That is, *backValid* and *backSliver* are both set true. In this case, there are no productive positions in this stamp column above the current position, but visiting the position above may still be necessary. If no positions more promising than moving *back* are found, then the sliver position above should be visited, as it may lead to productive positions in the next stamp column to the right. Figs. 13 and 14 show situations in which the position above is a sliver and therefore non-productive, but must be visited in order to get to the productive position diagonally up and to the right. In Fig. 13, the triangle 1310 satisfies this sliver condition for stamp 1300. There are no other valid positions, so the sliver will be visited immediately so that the stamp can reach the position diagonally up and right, which does contain a couple of fragments of triangle 1310. In Fig. 14, the triangle 1410 satisfies this sliver condition for stamp 1400, but also satisfies one of the *forward* sliver conditions discussed below. The stamp will save the position above in the *backSave* stamp context, then visit the *forward* position below. Since that path does not lead to any valid *over* positions, whether productive or not, the saved sliver context will be restored, and the stamp will visit the unproductive *back* position above in order to visit the productive position diagonally up and right.

20

25

30

35

Similarly, if the stamp is moving *over* from right to left, and the object does not intersect (*UL*, *UR*), but does intersect (*LT*, *UL*), the *back* position is again marked as a valid, but sliver, position.

5 Finally, if the stamp is moving *over* from right to left, and the object does not intersect (*UL*, *UR*), but does intersect (*UR*, *RT*), and the stamp is in the first (right-most) column of the object (that is, it's *x* coordinate is the same as the starting stamp position's *x* coordinate), the *back* position is marked as a sliver. In this special case, there may be productive positions within this column stampline. In Fig. 15, the triangle 1510 satisfies this special
10 sliver condition for stamp 1500. The stamp 1500 will move up, again trigger this special sliver condition, and finally move up again before reaching a productive position in which a sample point is inside the triangle.

The reasons that this special sliver case is enabled only for the first column of the object are
15 subtle. In the stamp columns between the first (right-most) and last (left-most) stamp columns (if any), it does not matter if the special case is enabled: both algorithms described below traverse the object in a fashion that avoids moving *back* due to a (*UR*, *RT*) intersection. In these middle columns, the stamp visits positions "outside" to the right of the object, which allows it to use the *LT* and *LB* probes to move *over* at the most appropriate
20 time. However, this special sliver case must be disabled in the last (left-most) column of any object that contains two or more column stamplines, so that the stamp does not visit positions in the last column that are guaranteed to be unproductive. Fig. 16 shows an example in which the stamp 1605 started at the lower right vertex, and has already visited stamp positions 1 through 4 (1601-1604). Though the current stamp position 1605 has a
25 (*UR*, *RT*) intersection, there is no point in going up, as all positions above in triangle 1610 are unproductive. The special sliver enabling is controlled by the state bit *firstColumnInObject*, which is initialized true, but set false by the movement algorithm immediately upon visiting an *over* position.

30 If none of the above conditions are met, the stamp never need move up from the current position, and both *backValid* and *backSliver* are set false.

Determining *forwardValid* and *forwardSliver*

35 Although the probe points along the bottom of the stamp are not placed symmetrically with the probes at the top of the stamp, the conditions for moving down are symmetrical with the

conditions for moving up. Rather than testing if the object intersects the segment (*LL*, *RB*) as does prior art, the present embodiment instead tests for intersection with the smaller segment (*LL*, *LR*), as well as one or both of the small horizontal segments (*LR*, *RB*) and (*LB*, *LL*).

If the object intersects (*LL*, *LR*), the position below may be productive: *forwardValid* is set true, and *forwardSliver* is set false.

If the stamp is moving *over* from left to right, and the object does not intersect (*LL*, *LR*), but does intersect (*LR*, *RB*), the *forward* position is marked as a sliver: *forwardValid* and *forwardSliver* are set true.

Similarly, if the stamp is moving *over* from right to left, and the object does not intersect (*LL*, *LR*), but does intersect (*LB*, *LL*), the *forward* position is again marked as a sliver.

Finally, if the stamp is moving *over* from right to left, and the object does not intersect (*LL*, *LR*), but does intersect (*LR*, *RB*), and the stamp is in the first (right-most) column of the object (that is, it's *x* coordinate is the same as the starting stamp position's *x* coordinate), the *forward* position is marked as a sliver.

If none of the above conditions is met, the stamp never needs to move down from the current position, and both *forwardValid* and *forwardSilver* are set false.

Determining *overValid* and *overProductive*

If the stamp is moving *over* from left to right, *overValid* is set true if the object intersects (*RT*, *RB*). Efficiency can be further improved by determining if the *over* position is guaranteed to be productive. The invention may examine several different *over* positions in the next stampline while moving in the *forward* or *back* directions in the current stampline. By moving to an *over* position that is know to be productive when all *forward* and *back* moves have been made, the invention sometimes avoids moving to an unproductive top-most stamp position in the next column stampline. By moving to a lower position in the next stampline, the stamp may avoid such unproductive positions if they are invalid *back* moves from the position immediately below.

In this embodiment, it is assumed that the origin of the stamp is also a sample point. Further the present embodiment sets *overProductive* true if the stamp is moving *over* from left to right, *overValid* is true, and the probe *RT* is inside the object. A different, similar test could be used if the origin is not a sample point, such as in some antialiased sample point arrangements.

The tests for moving *over* from right to left are symmetrical: *overValid* is set true if the object intersects (*LT*, *LB*), and *overProductive* is set true if *overValid* is true and the probe *LT* is inside the object.

Exemplary C++ code for determining the sparse context *valid*, *sliver*, and *productive* bits is shown in Appendix A.

Step 1708: Quash *forward* and *back* slivers

If the sparse context *over*, or the saved context *overSave*, is valid, then there is no point in moving to any sliver *back* and *forward* positions, whether sparse or saved contexts, even if the *over* context is not known to be productive. The *back* and *forward* slivers never lead to any productive stamp positions in the current stampline, but merely lead to a valid stamp position in the next stampline *over*. Thus, a valid *over* position immediately quashes any sliver *back* and *forward* positions by setting their *valid* and *sliver* bits to false. Sliver quashing substantially simplifies the implementation of the desired movement priorities in steps 1709 and 1710.

Exemplary C++ code for step 1708 is in Appendix B.

Step 1709: Determine next stamp position and move stamp

Appendix C gives the C++ code for moving a stamp as performed by Fig. 17A, step 1709. As shown in Appendix C, stamp movement depends, at least in part, on the various previously determined valid bits (e.g., *forwardValid*, *backSaveValid*, etc.). In some instances, the next position of the fragment stamp is restored from one of the various saved context values (e.g., *backSave*, *overSave*, etc.).

In particular, the C++ code of Appendix C, in conjunction with the sliver quashing of step 1708 shown in Appendix B, implements a stamp movement algorithm where the fragment

stamp moves to an unvisited non-sliver valid *forward* stamp position or an unvisited non-sliver valid *back* stamp position, if any such stamp position exists. If not, the fragment stamp attempts to move to an unvisited valid *over* stamp position, if any such stamp position exists. If the unvisited valid *over* stamp position is unavailable, then the fragment stamp attempts to move to an unvisited sliver *forward* stamp position or an unvisited sliver *back* stamp position, if any such stamp position exists. The sliver quashing of step 1708 simplifies the C++ code of Appendix C, by eliminating the need to test for *forward* and *back* non-sliver positions separately from *forward* and *back* sliver positions..

Step 1710: Update saved contexts

After deciding which way to move, Fig. 17A, step 1710 decides which sparse contexts, if any, should be stored into the corresponding saved contexts, and which saved contexts should be invalidated. Although any given sparse context can be stored into exactly one of the saved contexts in this step, multiple different sparse contexts can be stored into multiple different saved contexts. The sparse *back* context may be stored into *backSave*, and/or *over* may be stored into *overSave*. In order to maintain the “first best” *over* position in *overSave*, an *over* position that is known to be productive replaces a previously saved *over* position that is not known to be productive.

Appendix D gives the C++ code for updating saved contexts as performed by step 1710.

Step 1711: Update other bookkeeping information

Finally, Fig. 17A, step 1711 updates the values that indicate in which direction the stamp is moving and other information, for example, if this is the first position within a stampline. The C++ code for this updating is given in Appendix E.

After completing step 1711, the process resumes at step 1707 to process the object at the current stamp position, until the movement logic of step 1709 determines that the object has been completely traversed.

Alternate embodiment - Unidirectional Movement in Stamplines

An alternative embodiment, shown in Fig. 17B, involves a different way of visiting positions within the object, as sketched in the section “Example of Order of Traversal Using

Unidirectional Movement in Stamlines” above. In the following sections, the meaning of the terms *forward*, *back*, and *over* are the same as above in the sense that *forward* will mean down, *back* will mean up, and *over* will mean left or right. However, this alternative embodiment gives *over* moves priority over *forward* or *back* moves, and so traverses row stamlines.

This alternative implementation can also paint column stamlines by transposing the position of the probes around the stamp, and by appropriately changing the derivation of the *forwardInsideObject*, *backInsideObject*, and *overInsideObject* bits as previously described.

This alternative implementation has no *overSave* stamp context, but instead has a *forwardSave* context. It saves the first valid position it finds below the stamlane in *forwardSave*, and the first valid position it finds above the stamlane in *backSave*. When it gets to the right (or left, if going right-to-left) edge of the object, it then restores the *forwardSave* position, and proceeds along that stamlane, again saving the first valid position it finds below the stamlane in *forwardSave*. When it reaches the bottom of the object, it restores the *backSave* position, proceeds along that stamlane, saving the first valid position it finds above the stamlane in *backSave*. When it reaches the top of the object and cannot restore a *backSave* position, it is done generating all fragments within the object.

The saved contexts represent the following positions:

1. *backSave*: the first best position not yet visited found above the current (row) stamlane. (“First best” means that the first non-sliver *back* position is saved, and a sliver *back* position is saved only if there is no valid *over* position to which to move.)
2. *forwardSave*: the first best position not yet visited found below the current stamlane.

In this alternative implementation, in Fig. 17B, steps 1701, and steps 1703 through 1705 are the same as those of Fig. 17A. Steps 1702b, and 1706b through 1711b, described below, are analogous to steps 1702, and 1706 through 1711 of Fig. 17A, but with the changes needed to implement the alternate traversal order.

Step 1702b: Select a starting vertex on a side of the bounding box.

Step 1702b is similar to Step 1702, except that the conditions for row and column stamplines are swapped. That is, if row stamplines are desired the starting vertex must be on the left or right edge of the minimal bounding box and the non-transposed probes are used; if column stamplines are desired the starting vertex must be on the top or bottom edge of the minimal bounding box and the transposed probes are used.

Step 1706b: Initialize the starting bookkeeping state of the invention

Step 1706b is similar to Step 1706, except *forwardSave* takes the place of *overSave*, and *firstStamplineInObject* takes the place of *firstPositionInLine*:

```
if ( !transposedProbes && x_start == bbox.xmin
    || transposedProbes && y_start == bbox.ymin)
    dirOver = POSITIVE
else
    dirOver = NEGATIVE;
dirTopToBottom = (!transposedProbes && y_start == bbox.ymin
    || transposedProbes && x_start == bbox.xmin) ;
dirForwardBack = POSITIVE;           // Stamp movement from row to row in the
                                     // forward or back direction?
firstColumnInObject = true;           // Haven't made an over move yet?
firstStamplineInObject = true;        // First stampline, where we save both forward
                                     // and back positions?

backSaveValid = false;
backSaveSliver = false;
forwardSaveValid = false;
forwardSaveSliver = false;
```

Step 1707b: Evaluate edge functions at each probe and compute valid and sliver bits

Step 1707b is nearly identical to step 1707 described above. However, an *over* position must be taken unless it is a sliver, and thus known to be unproductive and incapable of leading to a productive position in the stampline. Such a sliver is only followed as a last resort. Thus, this alternative implementation does not use the *overProductive* bit, and so it need not be computed in step 1707b. Instead, it computes *overSliver*, as shown in the following C++ code:

```
if (dirOver == NEGATIVE) {
    overValid = overInsideObject && leftIntersect;
    overSliver = !Intersects (LEFT_TOP, LEFT_MIDDLE);
} else {
```

```

        overValid = overInsideObject && rightIntersect;
        overSliver = !Intersects (RIGHT_TOP, RIGHT_MIDDLE);
    }
    overSliver &= overValid && dirTopToBottom;

```

Note that steps 1707b through 1711b are repeated until no further moves are possible in Step 1709b.

Step 1708b: Quash *forward* and *back* slivers

Step 1708b is nearly identical to Step 1708, except that *forwardSave* slivers must be quashed also if *over* is valid. The following C++ code shows the operation of step 1708b:

```

if (overValid) {
    // Invalidate all forward and back slivers, in both sparse and saved contexts
    if (forwardSliver) {
        forwardValid = forwardSliver = false;
    }
    if (forwardSaveSliver) {
        forwardSaveValid = forwardSaveSliver = false;
    }
    if (backSliver) {
        backValid = backSliver = false;
    }
    if (backSaveSliver) {
        backSaveValid = backSaveSliver = false;
    }
}

```

Step 1709b: Determine next stamp position

Here is where the alternate method differs substantially from the bi-directional movement algorithm. Note that giving *over* moves priority means that the invention cannot avoid moving to an *over* position that is not known to be productive. Further, *over* moves should not invalidate *back* or *forward* slivers that are not adjacent to the current stampline, as described in more detail below in step 1710b.

Appendix F contains C++ code that implements the uni-directional movement algorithm.

In particular, the C++ code of Appendix F implements an algorithm where the fragment stamp moves an unvisited valid non-sliver *over* stamp position, if any such stamp position exists.

Otherwise, the fragment stamp moves to an unvisited valid non-sliver *forward* or *back* stamp position, if the appropriate such stamp position exists, either adjacent to the current position or previously saved by step 1710b. A *forward* position is chosen only if the algorithm is moving from stampline to stampline in the *forward* direction; a *back* position is chosen only if the algorithm is moving from stampline to stampline in the *back* direction. The stampline to stampline movement is controlled in part by the *dirForwardBack* bookkeeping state bit.

Otherwise, the fragment stamp moves to an unvisited sliver *over* stamp position, if any such stamp position exists.

Otherwise, the fragment stamp moves to an unvisited sliver *forward* or *back* stamp position, if the appropriate such stamp position exists. Again, a *forward* position is chosen only if the algorithm is moving from stampline to stampline in the *forward* direction; a *back* position is chosen only if the algorithm is moving from stampline to stampline in the *back* direction.

Again, the sliver quashing of step 1708b substantially simplifies the C++ code in Appendix F, by removing the need to test non-sliver and sliver cases separately. Note also that the code in Appendix F anticipates when *dirForwardBack* will change to NEGATIVE, by moving directly to a *back* or *backSave* position when *dirForwardBack* is POSITIVE, but no valid *forward* or *forwardSave* position exists.

Step 1710b: Update saved contexts

In the first stampline of an object, we can save into one or both of *backSave* and *forwardSave*. After that, we can save into *forwardSave* if *dirForwardBack* = POSITIVE, else we can save into *backSave*. If any kind of *forward* move is made, either directly to the sparse *forward* context, or indirectly by loading the *forwardSave* context, the *backSaveSliver* bit is set false. Symmetrically, if any kind of *back* move is made, the *forwardSaveSliver* bit is set false. This prevents step 1708b from using an unrelated valid *over* position to invalidate a saved sliver position that may lead to a productive position. Appendix G contains C++ code to update the saved contexts.

Step 1711b: Update other bookkeeping information

Appendix H shows the C++ code to update the other bookkeeping information. Again, this is similar to Step 1711, except there is no *goOverSave* way of moving *over*, and *firstStamplineInObject* is updated differently from *firstPositionInLine*.

5

Graphics Accelerator Logic

Fig. 19 depicts a simplified representation of the graphics accelerator 108 (Fig. 1). An interface 1950, such as a PCI interface, couples the graphics accelerator to the system chipset 104 (Fig. 1). Graphics rendering and other commands received from the processing unit 102 (Fig. 1) via the system chipset are parsed by a command parser 1952. The parser 1952 determines the object rendering operations to be performed, and passes these to a fragment generator 1960, which will be described in more detail with reference to Figs. 20 and 21. Fragments generated by the fragment generator 1960 are further processed by a texture processor 1962 so as to apply a specified texture pattern, if any, to the object being rendered. The resulting fragments are then passed to one or more pixel pipelines 1922 for pixel processing and storage. A pixel pipeline 1922 preferably includes a pixel processor 1970 for storing fragments into the frame buffer 1972 via a memory controller 1974. The pixel processor 1970 may perform tasks such as combining a fragment previously stored in the frame buffer 1972 with a fragment obtained from the graphics accelerator 108 and then storing the resulting combined fragment back into the frame buffer 1972.

A video controller 1980 couples the frame buffer 1972 to a display device 126 (Fig. 1). The video controller 1980 reads pixel values from the frame buffer 1972 via memory controller 1974 and sends corresponding pixel information to the display device 126 for display.

25

Figs. 20 and 21 show some of the circuitry of the fragment generator 1960. Edge initialization logic 2010 generates or provides edge and edge increment values for the next object to be rendered that are stored in a set of next object edge registers 2020. These registers 2020 include next object *x* and *y* edge increment registers 2023, and next object initial edge values registers 2025. Next object registers ready flag 2027 is enabled when registers 2025 and 2023 contain valid edge and edge increment values, and when registers 2125 and 2123 (Fig. 21), described below, contain valid channel and channel increment values. After one primitive object has been rendered, when the next object registers ready flag 2027 is set, the next object *x* and *y* edge increments 2023 are loaded into the current *x* and *y* edge increments 2030, and the next object initial edge values 2025 are loaded into the current edge context 2045.

35

Each of the saved contexts (described above in detail for the various embodiments) includes a saved edge context, stored in registers 2040, and a saved channel context, stored in registers 2140 (Fig. 21). A data structure representation of an edge context is shown in Fig. 23, and includes four edge function values plus a valid flag and a sliver flag. (The *over* contexts contain a *productive* flag rather than a sliver flag for the first embodiment that moves bidirectionally within a stampline.) Each of the edge function values represents the evaluation of a half-plane edge function at the origin of a stamp location. A data structure representation of a channel context is shown in Fig. 24, and includes color, transparency, depth, fog, texture coordinate, texture derivative and *x* and *y* coordinate values.

The current context includes the current edge context, stored in registers 2045, and the current channel context, stored in registers 2145 (Fig. 21).

Each of the three sparse contexts *forward*, *back*, and *over* includes only an edge context. The sparse edge contexts are not stored in registers 2040, but are generated by edge evaluators 2050

The current edge context 2045 and the current *x* and *y* edge increments 2030 are conveyed to a set of edge evaluators 2050. The edge evaluators 2050 are used to determine whether various points in and around the current stamp fall within the object being rendered. Each edge evaluator contains circuitry for evaluating a set of half-plane edge functions, by adding appropriate multiples of the current *x* and *y* edge increments 2030 for each edge to the edge values from the current edge context 2040. Each set of edge evaluators together determine whether a given (*x*, *y*) position of a pixel or sample point is within a specified object. Edge evaluation is described above with respect to step 1707 and 1707b.

There are edge evaluators 2052 for evaluating the sample points in the current stamp, an edge evaluator 2054 for the origin of the current stamp, edge evaluators 2056 for evaluating probe points around the current stamp, as well as edge evaluators 2058 for evaluating speculative points around the current stamp, in particular, the origin of the stamp positions immediately to the left and above the current position.

Fig. 22 depicts exemplary edge evaluation locations for a 4 x 2 stamp. The locations marked by X's are sample points in the stamp, the location marked by a circle is the origin of the stamp, the locations marked by diamonds are probe points, and the locations marked

by a square are speculative points. The edge evaluators 2052 (Fig. 20) for all seven sample points, as well as the edge evaluator 2056 for probe point RB, compute only the sign bit of the half-plane functions, as these values are needed only for testing if points are inside the object being rendered. The edge evaluator 2058 for the speculative points SB and SO, as well as the edge evaluator 2056 for probe point LB and RT, compute the full half-plane function values, as these values may be loaded into the current edge context 2045 if the stamp movement decision circuitry 2060 moves the stamp *back*, *over* (when *dirOver* is NEGATIVE), *forward*, or *over* (when *dirOver* is POSITIVE), respectively. The origin edge evaluator 2054 passes the current edge context values 2045 through without modifying them.

The results generated by the origin and sample point edge evaluators 2054 and 2052, respectively, are used to form a fragment sample mask 2095. The mask is a set of bits indicating which sample points of the stamp fall within the object being rendered.

The results generated by the probe point edge evaluators 2056 are used by logic 2080 to compute valid, sliver, and productive bits. This logic is described above with respect to step 1707 and 1707b.

Stamp movement decision circuitry 2060 is described in detail above with respect to step 1709 and 1709b. This circuitry 2060 uses information from the saved edge contexts 2040, information from bookkeeping state circuitry 2070, and the valid, sliver, and productive bits from logic 2080. The stamp movement decision circuitry 2060 generates control signals for updating the saved edge contexts 2040, the current edge context 2045, and the bookkeeping state 2070. The stamp movement decision is also used by the channel context update circuitry shown in Fig. 21.

Channel initialization logic 2110 (Fig. 21) generates or provides initial channel and channel increment values that are stored in a set of next object channel registers 2120. These registers 2120 include next object *x* and *y* channel increment registers 2123, and next object initial channel value registers 2125. After one primitive object has been rendered, the next object *x* and *y* channel increments 2123 are loaded into the current *x* and *y* channel increments 2130, and the initial channel values 2125 are loaded into the current channel context 2045.

While the current edge context 2045 contains the edge values for the stamp origin during the current cycle, the current channel context 2145 contains the channel values for the stamp origin on the previous cycle. The current edge context 2045 is needed immediately in a cycle, so that the probe point edge evaluators 2056 can quickly deliver results to the stamp movement decision circuitry 2060. Between the probe point edge evaluators 2056 and the speculative point edge evaluators 2058, the edge functions are completely evaluated to their full precision for the four adjacent Manhattan stamp positions to which the stamp may immediately move. A similar arrangement for channels would require evaluating all channel values for these nearby positions, then multiplexing between these possibilities and the saved channel contexts using control signals from the stamp movement decision circuitry 2060. Unlike edge function values, channel values are not needed by stamp movement decision circuitry 2060, and so this arrangement wastefully evaluates channel values for positions that will not be moved to. To reduce the logic devoted to computing channel values, the logic in Fig. 20 delays computing channel values until *after* the stamp movement decision circuitry 2060 generates stable control signals. Allowing this channel computation to proceed in the same cycle in which the stamp movement decision is made would require increasing the cycle time of the logic shown in Figs. 20 and 21. Instead, the channel computation logic in Fig. 21 saves the control signals from stamp movement decision circuitry 2060 (Fig. 20) in decision result latch 2180, and computes the channel values during the next cycle.

The compute next channel values logic 2150 selects channel data from the current channel context 2145 if the decision result latch 2180 indicates a move to an adjacent position, or from one of the saved channel contexts in registers 2140 if the decision result latch 2180 indicates a move to a saved position. The compute next channel values logic 2150 adds the appropriate multiples of the *x* and *y* channel increments 2130 to the selected context to compute the interpolated channel values 2195 and to load into the current channel context 2145 for use in the next cycle. If the decision result latch 2180 indicates that one or more adjacent position should be saved, the current channel context 2145 is loaded into the appropriate contexts in saved channel contexts 2140.

This invention is described using specific terms and examples. It is to be understood that various other adaptations and modifications may be made within the spirit and scope of the invention. Therefore, it is the object of the appended claims to cover all such variations and modifications as come within the true spirit and scope of the invention.

APPENDIX A

```

leftIntersect = Intersects (LEFT_TOP, LEFT_BOTTOM);
rightIntersect = Intersects (RIGHT_TOP, RIGHT_BOTTOM);

```

```

5  if (dirOver == NEGATIVE) {
    overValid = overInsideObject && leftIntersect;
    overProductive = overValid && Intersects (LEFT_TOP, LEFT_TOP);
  } else {
    overValid = overInsideObject && rightIntersect;
    overProductive = overValid && Intersects (RIGHT_TOP, RIGHT_TOP);
  }

```

```

10 downIntersect = Intersects (LOWER_LEFT, LOWER_RIGHT);
    upIntersect = Intersects (UPPER_LEFT, UPPER_RIGHT);

```

```

    rightForwardSliver =
      (((dirOver == NEGATIVE && firstColumnInObject) || (dirOver ==
POSITIVE))
      && Intersects (LOWER_RIGHT, RIGHT_BOTTOM));

```

```

15 leftForwardSliver =
    (dirOver == NEGATIVE && Intersects (LEFT_BOTTOM, LOWER_LEFT));

```

```

    rightBackSliver =
      (((dirOver == NEGATIVE && firstColumnInObject)
|| (dirOver == POSITIVE)) && Intersects (UPPER_RIGHT, RIGHT_TOP));

```

```

20 leftBackSliver =
    (dirOver == NEGATIVE && Intersects (LEFT_TOP, UPPER_LEFT));

```

```

    forwardSliver = forwardInsideObject && (rightForwardSliver || leftForwardSliver)
      && !downIntersect;

```

```

    backSliver = backInsideObject && (rightBackSliver || leftBackSliver)
      && !upIntersect;

```

```

25 forwardValid = (forwardInsideObject && downIntersect) || forwardSliver;
    backValid = (backInsideObject && upIntersect) || backSliver;

```

30

35

APPENDIX B

```
if (overValid || overSaveValid) {  
    // Invalidate all forward and back slivers, in both sparse and saved contexts  
    if (forwardSliver) {  
5         forwardValid = forwardSliver = false;  
    }  
    if (backSliver) {  
        backValid = backSliver = false;  
    }  
    if (backSaveSliver) {  
        backSaveValid = backSaveSliver = false;  
10    }  
}
```

15
20

25

30

35

APPENDIX C

```

goForward = goBackSave = goBack = goOver = goOverSave = false;
if (forwardValid && (dirForwardBack == POSITIVE)) {
    // forward is valid and we're already moving forward in this stampline.
    goForward = true;
    current = forward;

} else if (backSaveValid) {
    // Saved back position, so go back to it and then move backward through
    stampline
    goBackSave = true;
    current = backSave;

} else if (backValid && (firstPositionInLine || (dirForwardBack == NEGATIVE)))
{
    // Back position valid, and either (1) we're at the first position in this
    // stampline (and no forward move), so we want to bypass directly to
    // back, or (2) we're already going backward in this stampline
    goBack = true;
    current = back;

} else if (overValid && (!overSaveValid || (!overSaveProductive &&
overProductive))) {
    // over valid, and either no saved over, or else over is superior to overSave.
    goOver = true;
    current = over;

} else if (overSaveValid) {
    // Saved over is valid, use it
    goOverSave = true;
    current = overSave;

} else {
    // We've generated all fragments within the object.
    Get new object and go to Step 1701;
}

```

APPENDIX D

```

// Any more positions to visit in this stampline?
moreInStampline = goForward || goBack || backSaveValid;

5 // Save back into backSave if it's valid, this is the first stamp position within
// the stampline, and we'll be moving forward
writeBackSave = backValid && firstPositionInLine && forwardValid;
if (goBackSave) {
    backSaveValid = false;
    backSaveSliver = false;
} else if (writeBackSave) {
    backSave = back;
10    backSaveValid = true;
    backSaveSliver = backSliver;
}

// Save first over position found into overSave as long as we still have other
// positions to visit in this stampline (that is, we don't take the goOver bypass).
// Also allow replacement of overSave if it is not known productive and over is.
writeOverSave = overValid && moreInStampline
15    && (!overSaveValid || (!overSaveProductive && overProductive));
if (goOverSave) {
    overSaveValid = false;
    overSaveProductive = false;
} else if (writeOverSave) {
    overSave = over;
    overSaveValid = true;
    overSaveProductive = overProductive;
20 }

```

25

30

35

APPENDIX E

```
// Moving in forward direction in stampline unless we choose one of the two  
// back moves
```

```
dirForwardBack = ((goBack || goBackSave) ? NEGATIVE : POSITIVE);
```

5

```
// If we moved over, we're at the first stamp position within a stampline  
firstPositionInLine = (goOver || goOverSave);
```

```
// If we moved over, we're also no longer in the first column of the object,  
// and the special sliver test should be disabled.
```

```
if (firstPositionInLine) {  
    firstColumnInObject = false;
```

10

```
}
```

```
goto Step 1707;
```

15

20

25

30

35

APPENDIX F

```

5      goForward = goForwardSave = goBack = goBackSave = goOver = false;

      if (overValid && !overSliver) {
          // over valid and not a sliver
          goOver = true;
          current = over;

      } else if (forwardSaveValid) {
10         // Use the saved position
          goForwardSave = true;
          current = forwardSave;

      } else if (forwardValid && dirForwardBack == POSITIVE) {
          // Use the bypass to move directly to forward position
          goForward = true;
          current = forward;
15      } else if (backSaveValid) {
          // Saved back position, so go back to it
          goBackSave = true;
          current = backSave;

      } else if (backValid
20         && (firstStamplineInObject || (dirForwardBack == NEGATIVE))) {
          // Back position valid, and either (1) we're on the first
          // stampline (and no forward move), so we want to bypass directly to
          // back, or (2) we're already going backward in this stampline
          goBack = true;
          current = back;

      } else if (overValid) {
25         // This is a sliver, but didn't find anything better to do, and we might find a
          // good forward position eventually if we go over.
          goOver = true;
          current = over;

      } else {
30         load new object and go to Step 1701; // Nothing left to do in this object
      }

```

35

APPENDIX G

```
writeBackSave = backValid && !backSaveValid && !goBack
    && (firstStamplineInObject || dirForwardBack == NEGATIVE);
```

```
5  if (goBackSave) {
    backSaveValid = false;
    backSaveSliver = false;
  } else if (writeBackSave) {
    backSave = back;
    backSaveValid = true;
    backSaveSliver = backSliver;
  }
```

```
10 writeForwardSave = forwardValid && !forwardSaveValid && !goForward
    && dirForwardBack == POSITIVE;
```

```
15 if (goForwardSave) {
    forwardSaveValid = false;
    forwardSaveSliver = false;
  } else if (writeForwardSave) {
    forwardSave = forward;
    forwardSaveValid = true;
    forwardSaveSliver = forwardSliver;
  }
```

```
20 if (goForward || goForwardSave) {
    backSaveSliver = false;
  } else if (goBack || goBackSave) {
    forwardSaveSliver = false;
  }
}
```

25

30

35

APPENDIX H

```
// Moving in forward direction in stampline unless we choose one of the two
// back moves
dirForwardBack = ((goBack || goBackSave) ? NEGATIVE : POSITIVE);

5   if (goOver) {
    // If we moved over, we're also no longer in the first column of the object,
    // and the special sliver test should be disabled.
    firstColumnInObject = false;
  } else {
    // Otherwise we are no longer on the first stampline
    firstStamplineInObject = false;
10  }
    goto Step 1707b;
```

11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99
100
101
102
103
104
105
106
107
108
109
110
111
112
113
114
115
116
117
118
119
120
121
122
123
124
125
126
127
128
129
130
131
132
133
134
135
136
137
138
139
140
141
142
143
144
145
146
147
148
149
150
151
152
153
154
155
156
157
158
159
160
161
162
163
164
165
166
167
168
169
170
171
172
173
174
175
176
177
178
179
180
181
182
183
184
185
186
187
188
189
190
191
192
193
194
195
196
197
198
199
200
201
202
203
204
205
206
207
208
209
210
211
212
213
214
215
216
217
218
219
220
221
222
223
224
225
226
227
228
229
230
231
232
233
234
235
236
237
238
239
240
241
242
243
244
245
246
247
248
249
250
251
252
253
254
255
256
257
258
259
260
261
262
263
264
265
266
267
268
269
270
271
272
273
274
275
276
277
278
279
280
281
282
283
284
285
286
287
288
289
290
291
292
293
294
295
296
297
298
299
300
301
302
303
304
305
306
307
308
309
310
311
312
313
314
315
316
317
318
319
320
321
322
323
324
325
326
327
328
329
330
331
332
333
334
335
336
337
338
339
340
341
342
343
344
345
346
347
348
349
350
351
352
353
354
355
356
357
358
359
360
361
362
363
364
365
366
367
368
369
370
371
372
373
374
375
376
377
378
379
380
381
382
383
384
385
386
387
388
389
390
391
392
393
394
395
396
397
398
399
400
401
402
403
404
405
406
407
408
409
410
411
412
413
414
415
416
417
418
419
420
421
422
423
424
425
426
427
428
429
430
431
432
433
434
435
436
437
438
439
440
441
442
443
444
445
446
447
448
449
450
451
452
453
454
455
456
457
458
459
460
461
462
463
464
465
466
467
468
469
470
471
472
473
474
475
476
477
478
479
480
481
482
483
484
485
486
487
488
489
490
491
492
493
494
495
496
497
498
499
500
501
502
503
504
505
506
507
508
509
510
511
512
513
514
515
516
517
518
519
520
521
522
523
524
525
526
527
528
529
530
531
532
533
534
535
536
537
538
539
540
541
542
543
544
545
546
547
548
549
550
551
552
553
554
555
556
557
558
559
560
561
562
563
564
565
566
567
568
569
570
571
572
573
574
575
576
577
578
579
580
581
582
583
584
585
586
587
588
589
590
591
592
593
594
595
596
597
598
599
600
601
602
603
604
605
606
607
608
609
610
611
612
613
614
615
616
617
618
619
620
621
622
623
624
625
626
627
628
629
630
631
632
633
634
635
636
637
638
639
640
641
642
643
644
645
646
647
648
649
650
651
652
653
654
655
656
657
658
659
660
661
662
663
664
665
666
667
668
669
670
671
672
673
674
675
676
677
678
679
680
681
682
683
684
685
686
687
688
689
690
691
692
693
694
695
696
697
698
699
700
701
702
703
704
705
706
707
708
709
710
711
712
713
714
715
716
717
718
719
720
721
722
723
724
725
726
727
728
729
730
731
732
733
734
735
736
737
738
739
740
741
742
743
744
745
746
747
748
749
750
751
752
753
754
755
756
757
758
759
760
761
762
763
764
765
766
767
768
769
770
771
772
773
774
775
776
777
778
779
780
781
782
783
784
785
786
787
788
789
790
791
792
793
794
795
796
797
798
799
800
801
802
803
804
805
806
807
808
809
810
811
812
813
814
815
816
817
818
819
820
821
822
823
824
825
826
827
828
829
830
831
832
833
834
835
836
837
838
839
840
841
842
843
844
845
846
847
848
849
850
851
852
853
854
855
856
857
858
859
860
861
862
863
864
865
866
867
868
869
870
871
872
873
874
875
876
877
878
879
880
881
882
883
884
885
886
887
888
889
890
891
892
893
894
895
896
897
898
899
900
901
902
903
904
905
906
907
908
909
910
911
912
913
914
915
916
917
918
919
920
921
922
923
924
925
926
927
928
929
930
931
932
933
934
935
936
937
938
939
940
941
942
943
944
945
946
947
948
949
950
951
952
953
954
955
956
957
958
959
960
961
962
963
964
965
966
967
968
969
970
971
972
973
974
975
976
977
978
979
980
981
982
983
984
985
986
987
988
989
990
991
992
993
994
995
996
997
998
999
1000
1001
1002
1003
1004
1005
1006
1007
1008
1009
1010
1011
1012
1013
1014
1015
1016
1017
1018
1019
1020
1021
1022
1023
1024
1025
1026
1027
1028
1029
1030
1031
1032
1033
1034
1035
1036
1037
1038
1039
1040
1041
1042
1043
1044
1045
1046
1047
1048
1049
1050
1051
1052
1053
1054
1055
1056
1057
1058
1059
1060
1061
1062
1063
1064
1065
1066
1067
1068
1069
1070
1071
1072
1073
1074
1075
1076
1077
1078
1079
1080
1081
1082
1083
1084
1085
1086
1087
1088
1089
1090
1091
1092
1093
1094
1095
1096
1097
1098
1099
1100
1101
1102
1103
1104
1105
1106
1107
1108
1109
1110
1111
1112
1113
1114
1115
1116
1117
1118
1119
1120
1121
1122
1123
1124
1125
1126
1127
1128
1129
1130
1131
1132
1133
1134
1135
1136
1137
1138
1139
1140
1141
1142
1143
1144
1145
1146
1147
1148
1149
1150
1151
1152
1153
1154
1155
1156
1157
1158
1159
1160
1161
1162
1163
1164
1165
1166
1167
1168
1169
1170
1171
1172
1173
1174
1175
1176
1177
1178
1179
1180
1181
1182
1183
1184
1185
1186
1187
1188
1189
1190
1191
1192
1193
1194
1195
1196
1197
1198
1199
1200
1201
1202
1203
1204
1205
1206
1207
1208
1209
1210
1211
1212
1213
1214
1215
1216
1217
1218
1219
1220
1221
1222
1223
1224
1225
1226
1227
1228
1229
1230
1231
1232
1233
1234
1235
1236
1237
1238
1239
1240
1241
1242
1243
1244
1245
1246
1247
1248
1249
1250
1251
1252
1253
1254
1255
1256
1257
1258
1259
1260
1261
1262
1263
1264
1265
1266
1267
1268
1269
1270
1271
1272
1273
1274
1275
1276
1277
1278
1279
1280
1281
1282
1283
1284
1285
1286
1287
1288
1289
1290
1291
1292
1293
1294
1295
1296
1297
1298
1299
1300
1301
1302
1303
1304
1305
1306
1307
1308
1309
1310
1311
1312
1313
1314
1315
1316
1317
1318
1319
1320
1321
1322
1323
1324
1325
1326
1327
1328
1329
1330
1331
1332
1333
1334
1335
1336
1337
1338
1339
1340
1341
1342
1343
1344
1345
1346
1347
1348
1349
1350
1351
1352
1353
1354
1355
1356
1357
1358
1359
1360
1361
1362
1363
1364
1365
1366
1367
1368
1369
1370
1371
1372
1373
1374
1375
1376
1377
1378
1379
1380
1381
1382
1383
1384
1385
1386
1387
1388
1389
1390
1391
1392
1393
1394
1395
1396
1397
1398
1399
1400
1401
1402
1403
1404
1405
1406
1407
1408
1409
1410
1411
1412
1413
1414
1415
1416
1417
1418
1419
1420
1421
1422
1423
1424
1425
1426
1427
1428
1429
1430
1431
1432
1433
1434
1435
1436
1437
1438
1439
1440
1441
1442
1443
1444
1445
1446
1447
1448
1449
1450
1451
1452
1453
1454
1455
1456
1457
1458
1459
1460
1461
1462
1463
1464
1465
1466
1467
1468
1469
1470
1471
1472
1473
1474
1475
1476
1477
1478
1479
1480
1481
1482
1483
1484
1485
1486
1487
1488
1489
1490
1491
1492
1493
1494
1495
1496
1497
1498
1499
1500
1501
1502
1503
1504
1505
1506
1507
1508
1509
1510
1511
1512
1513
1514
1515
1516
1517
1518
1519
1520
1521
1522
1523
1524
1525
1526
1527
1528
1529
1530
1531
1532
1533
1534
1535
1536
1537
1538
1539
1540
1541
1542
1543
1544
1545
1546
1547
1548
1549
1550
1551
1552
1553
1554
1555
1556
1557
1558
1559
1560
1561
1562
1563
1564
1565
1566
1567
1568
1569
1570
1571
1572
1573
1574
1575
1576
1577
1578
1579
1580
1581
1582
1583
1584
1585
1586
1587
1588
1589
1590
1591
1592
1593
1594
1595
1596
1597
1598
1599
1600
1601
1602
1603
1604
1605
1606
1607
1608
1609
1610
1611
1612
1613
1614
1615
1616
1617
1618
1619
1620
1621
1622
1623
1624
1625
1626
1627
1628
1629
1630
1631
1632
1633
1634
1635
1636
1637
1638
1639
1640
1641
1642
1643
1644
1645
1646
1647
1648
1649
1650
1651
1652
1653
1654
1655
1656
1657
1658
1659
1660
1661
1662
1663
1664
1665
1666
1667
1668
1669
1670
1671
1672
1673
1674
1675
1676
1677
1678
1679
1680
1681
1682
1683
1684
1685
1686
1687
1688
1689
1690
1691
1692
1693
1694
1695
1696
1697
1698
1699
1700
1701
1702
1703
1704
1705
1706
1707
1708
1709
1710
1711
1712
1713
1714
1715
1716
1717
1718
1719
1720
1721
1722
1723
1724
1725
1726
1727
1728
1729
1730
1731
1732
1733
1734
1735
1736
1737
1738
1739
1740
1741
1742
1743
1744
1745
1746
1747
1748
1749
1750
1751
1752
1753
1754
1755
1756
1757
1758
1759
1760
1761
1762
1763
1764
1765
1766
1767
1768
1769
1770
1771
1772
1773
1774
1775
1776
1777
1778
1779
1780
1781
1782
1783
1784
1785
1786
1787
1788
1789
1790
1791
1792
1793
1794
1795
1796
1797
1798
1799
1800
1801
1802
1803
1804
1805
1806
1807
1808
1809
1810
1811
1812
1813
1814
1815
1816
1817
1818
1819
1820
1821
1822
1823
1824
1825
1826
1827
1828
1829
1830
1831
1832
1833
1834
1835
1836
1837
1838
1839
1840
1841
1842
1843
1844
1845
1846
1847
1848
1849
1850
1851
1852
1853
1854
1855
1856
1857
1858
1859
1860
1861
1862
1863
1864
1865
1866
1867
1868
1869
1870
1871
1872
1873
1874
1875
1876
1877
1878
1879
1880
1881
1882
1883
1884
1885
1886
1887
1888
1889
1890
1891
1892
1893
1894
1895
1896
1897
1898
1899
1900
1901
1902
1903
1904
1905
1906
1907
1908
1909
1910
1911
1912
1913
1914
1915
1916
1917
1918
1919
1920
1921
1922
1923
1924
1925
1926
1927
1928
1929
1930
1931
1932
1933
1934
1935
1936
1937
1938
1939
1940
1941
1942
1943
1944
1945
1946
1947
1948
1949
1950
1951
1952
1953
1954
1955
1956
1957
1958
1959
1960
1961
1962
1963
1964
1965
1966
1967
1968
1969
1970
1971
1972
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019
2020
2021
2022
2023
2024
2025
2026
2027
2028
2029
2030
2031
2032
2033
2034
2035
2036
2037
2038
2039
2040
2041
2042
2043
2044
2045
2046
2047
2048
2049
2050
2051
2052
2053
2054
2055
2056
2057
2058
2059
2060
2061
2062
2063
2064
2065
2066
2067
2068
2069
2070
2071
2072
2073
2074
2075
2076
2077
2078
2079
2080
2081
2082
2083
2084
2085
2086
2087
2088
2089
2090
2091
2092
2093
2094
2095
2096
2097
2098
2099
2100
2101
2102
2103
2104
2105
2106
2107
2108
2109
2110
2111
2112
2113
2114
2115
2116
2117
2118
2119
2120
2121
2122
2123
2124
2125
2126
2127
2128
2129
2130
2131
2132
2133
2134
2135
2136
2137
2138
2139
2140
2141
2142
2143
2144
2145
2146
2147
2148
2149
2150
2151
2152
2153
2154
2155
2156
2157
2158
2159
2160
2161
2162
2163
2164
2165
2166
2167
2168
2169
2170
2171
2172
2173
2174
2175
2176
2177
2178
2179
2180
2181
2182
2183
2184
2185
2186
2187
2188
2189
2190
2191
2192
2193
2194
2195
2196
2197
2198
2199
2200
2201
2202
2203
2204
2205
2206
2207
2208
2209
2210
2211
2212
2213
221